$$\frac{\partial^2 (\Delta n_p)}{\partial x^2} - \frac{\Delta n_p}{L_n} = 0$$

$$\Delta n_p (W_p) = n_{po} \left[\exp\left(\frac{qV_F}{kT} \right) - 1 \right]$$

$$\Delta n_p (X_p) = 0$$

$$\Delta n_p (x) = A \exp\left(-\frac{x}{L_n} \right) + B \exp\left(\frac{x}{L_n} \right)$$

$$= A' \exp\left(-\frac{x - W_p}{L_n} \right) + B' \exp\left(\frac{x - W_p}{L_n} \right)$$

$$= A'' \cosh\left(\frac{x - W_p}{L_n} \right) + B'' \sinh\left(\frac{x - W_p}{L_n} \right)$$

$$\cosh(u) = \frac{1}{2}\left(e^{u} + e^{-u}\right)$$

$$\sinh(u) = \frac{1}{2}\left(e^{u} - e^{-u}\right)$$

$$\cosh(0) = 1$$

$$\sinh(0) = 0$$

$$\cosh(-u) = \cosh(u)$$

$$\sinh(-u) = -\sinh(u)$$

$$\frac{d}{dx}\left[\cosh(u)\right] = \sinh(u)$$

$$\frac{d}{dx}\left[\sinh(u)\right] = \cosh(u)$$

$$\cosh(u) \approx 1 + u^{2}$$

$$\sinh(u) \approx u$$

$$\cosh(u) \approx \frac{1}{2}e^{u}$$

$$\sinh(u) \approx \frac{1}{2}e^{u}$$

$$\left[\cosh(u)\right]^{2} - \left[\sinh(u)\right]^{2} = 1$$

$$\sinh(u - v) = \left[\sinh(u)\right]\left[\cosh(v)\right] - \left[\cosh(u)\right]\left[\sinh(v)\right]$$

Note the similarity with sine and cosine

$$\Delta n_p\left(W_p\right) = A'' \cosh\left(0\right) + B' \sinh\left(0\right) = n_{po}\left[\exp\left(\frac{qV_F}{kT}\right) - 1\right]$$

$$A'' = n_{po}\left[\exp\left(\frac{qV_F}{kT}\right) - 1\right] = \Delta n_{inj}$$

$$\Delta n_p\left(X_p\right) = \Delta n_{inj}\cosh\left(\frac{X_p - W_p}{L_n}\right) + B'' \sinh\left(\frac{X_p - W_p}{L_n}\right) = 0$$

$$B'' = -\Delta n_{inj}\frac{\cosh\left(\frac{X_p - W_p}{L_n}\right)}{\sinh\left(\frac{X_p - W_p}{L_n}\right)}$$

$$\Delta n_p\left(x\right) = \Delta n_{inj}\left[\cosh\left(\frac{x - W_p}{L_n}\right) - \frac{\cosh\left(\frac{X_p - W_p}{L_n}\right)}{\sinh\left(\frac{X_p - W_p}{L_n}\right)}\sinh\left(\frac{x - W_p}{L_n}\right)\right]$$

Yikes!

$$\Delta n_p(x) = \frac{\Delta n_{inj}}{\sinh\left(\frac{X_p - W_p}{L_n}\right)}\left[\sinh\left(\frac{X_p - W_p}{L_n}\right)\cosh\left(\frac{x - W_p}{L_n}\right) - \cosh\left(\frac{X_p - W_p}{L_n}\right)\sinh\left(\frac{x - W_p}{L_n}\right)\right]$$

Using the difference trig identity.

$$\Delta n_p(x) = \Delta n_{inj}\frac{\sinh\left(\frac{X_p - x}{L_n}\right)}{\sinh\left(\frac{X_p - W_p}{L_n}\right)}$$

general solution

If $L_n >> X_p - W_p$ (also means that $L_n >> X_p - x$)

$$\Delta n_p(x) = \Delta n_{inj}\frac{X_p - x}{X_p - W_p}$$

short diode

If $L_n << X_p - W_p$ (also $<< X_p - x$)

$$\Delta n_p(x) \approx \Delta n_{inj}\frac{\exp\left(\frac{X_p - x}{L_n}\right)}{\exp\left(\frac{X_p - W_p}{L_n}\right)}$$

$$= \Delta n_{inj}\exp\left(-\frac{x - W_p}{L_n}\right)$$

long diode

$$J_n\left(x\right) = qD_n \frac{\partial\left[\Delta n_p\left(x\right)\right]}{\partial x}$$

$$= -\frac{qD_n n_{po}}{L_n}\left[\frac{\cosh\left(\frac{X_p - x}{L_n}\right)}{\sinh\left(\frac{X_p - W_p}{L_n}\right)}\right]\left[\exp\left(\frac{qV_F}{kT}\right) - 1\right]$$

$$J_n\left(W_p\right) = -\frac{qD_n n_{po}}{L_n}\left[\frac{\cosh\left(\frac{X_p - W_p}{L_n}\right)}{\sinh\left(\frac{X_p - W_p}{L_n}\right)}\right]\left[\exp\left(\frac{qV_F}{kT}\right) - 1\right]$$

$$= -J_{SN}\left[\exp\left(\frac{qV_F}{kT}\right) - 1\right]$$