MOS threshold voltage

A reasonable definition of the onset of inversion is that the electron concentration underneath gate oxide becomes equal to the hole concentration in the bulk (far away from the gate). (The gate bias induces the semiconductor to become as n-type as it was p-type under flat-band condtions.) In terms of the band band diagram, this can be expressed as

$$q\phi_s = 2\left(E_i - E_{Fp}\right)\big|_{bulk}$$

$$q\phi_s = 2kT\ln\left(\frac{p_{po}}{n_i}\right) = 2kT\ln\left(\frac{N_A}{n_i}\right)$$

Then

$$qV_T = qV_{FB} + q\phi_{ox} + 2kT\ln\left(\frac{N_A}{n_i}\right)$$

We still need to determine the band-bending in the oxide, ϕ_{ox}.

To find oxide, ϕ_{ox}, we first relate the potential change to the electric field in the oxide.

$$-\frac{d\phi_{ox}}{dx} = \mathcal{E}_{ox}$$

We will assume (for now) that their no charge within the oxide or at the silicon/oxide interface. In that case,

$$\frac{d\mathcal{E}_{ox}}{dx} = 0$$

and the electric field must be constant. Then,

$$\phi_{ox} = \mathcal{E}_{ox} t_{ox}$$

However, we are still left with determining the electric field in the oxide. We do know that, at the interface, the electric fields must match according to the usual boundary conditions.

$$\varepsilon_{ox}\mathcal{E}_{ox} = \varepsilon_s \mathcal{E}_s$$

But we just kicking the can down the road — we have gone from needing to know ϕ_{ox} to needing to know the electric field in the oxide to needing to know the electric field in the semiconductor.

As a first cut, we can come up with an approximation for the electric field in the semiconductor. We know that positive voltage on the gate, with, the p-type semiconductor was depleted, which is very much akin to the depletion layer in a p-n junction. So we could use the electrostatics relationship in a depletion layer here.

$$\mathcal{E}_s(0) = \frac{2\phi_s}{w_p} \quad \text{and} \quad \phi_s = \frac{qN_A w_p^2}{2\varepsilon_s}$$

putting it together

$$\mathcal{E}_s(0) = \sqrt{\frac{2qN_A\phi_s}{\varepsilon_s}}$$

Now, unwinding it all

$$\mathcal{E}_{ox}(0) = \frac{\varepsilon_s}{\varepsilon_{ox}} \mathcal{E}_s(0) = \frac{1}{\varepsilon_{ox}} \sqrt{2q\varepsilon_s N_A \phi_s}$$

$$\phi_{ox} = \mathcal{E}_{ox}(0)\, t_{ox} = \frac{t_{ox}}{\varepsilon_{ox}} \sqrt{2q\varepsilon_s N_A \phi_s}$$

Since we are looking at the specific case of the onset of inversion, we know that

$$q\phi_s = 2kT \ln\left(\frac{N_A}{n_i}\right)$$

and then

$$\phi_{ox} = \frac{t_{ox}}{\varepsilon_{ox}} \sqrt{4\varepsilon_s N_A 2kT \ln\left(\frac{N_A}{n_i}\right)}$$

Finally, we have an (approximate) expression for the threshold voltage

$$V_{Tn} = V_{FB} + \frac{2kT}{q} \ln\left(\frac{N_A}{n_i}\right) + \frac{t_{ox}}{\varepsilon_{ox}} \sqrt{4\varepsilon_s N_A kT \ln\left(\frac{N_A}{n_i}\right)}$$

The reason that this is an approximation is that we used the expression for a depletion layer to determine the electric field in the semiconductor. But at the onset of inversion, the region is not truly depleted — electrons are beginning to pile up underneath the gate. We are ignoring those electrons, although we realize that they might well have some effect on the field.

From the threshold voltage expression, we see that there are two things that we can adjust to determine the value of threshold — the *p*-type doping in the semiconductor (increasing the doping increases the threshold voltage) and the oxide thickness (increasing oxide thickness also increases threshold voltage).

example calculation.

An NMOS cap structure where the gate is made of n^+ polysilicon has $N_A = 10^{16}\ cm^{-3}$ and t_{ox} = 50 nm. What is the threshold voltage?

First, find the flat-band voltage. For silicon / SiO_2 / n+ poly, the difference in the barrier heights is –0.03 eV and E_G = 1.12 eV.

$$qV_{FB} = q\Phi_M - q\mathrm{X}_S - E_G + \delta_p$$

$$\delta_p = kT \ln\left[\frac{N_V}{N_A}\right] = (0.0258\ \mathrm{eV}) \ln\left[\frac{1.0 \times 10^{19}\ \mathrm{cm}^{-3}}{10^{16}\ \mathrm{cm}^{-3}}\right] = 0.178\ \mathrm{eV}$$

$$qV_{FB} = -0.03\ \mathrm{eV} - 1.12\ \mathrm{eV} + 0.178\ \mathrm{eV} = -0.972\ \mathrm{eV}$$

(V_{FB} = –0.972 V)

$$\frac{kT}{q} \ln\left[\frac{N_A}{N_i}\right] = (0.0258\ \mathrm{V}) \ln\left[\frac{10^{16}\ \mathrm{cm}^{-3}}{6 \times 10^{9}\ \mathrm{cm}^{-3}}\right] = 0.370\ \mathrm{V}$$

$$\sqrt{4\varepsilon_s N_A kT \ln\left(\frac{N_A}{n_i}\right)}$$

$$= \sqrt{4\,(11.7)\left(8.85 \times 10^{-14}\,\frac{\text{F}}{\text{cm}}\right)(10^{16}\ \text{cm}^{-3})\left(1.6 \times 10^{-19}\,\frac{\text{J}}{\text{eV}}\right)(0.370\ \text{eV})}$$

$$= 4.95 \times 10^{-8}\,\frac{\text{C}}{\text{cm}^2}$$

$$V_{TN} = -0.972\ \text{V} + 2\,(0.370\ \text{V}) + \frac{25 \times 10^{-7}\ \text{cm}}{(3.9)\left(8.85 \times 10^{-14}\,\frac{\text{F}}{\text{cm}}\right)}\left(4.95 \times 10^{-8}\,\frac{\text{C}}{\text{cm}^2}\right)$$

$$= 0.127\ \text{V}$$