Moll-Ross

In forward-active operation for an *npn* BJT the boundary conditions are:

$$n_p(0) = n_{po} \exp\left(\frac{qV_{BE}}{kT}\right) \qquad n_p\left(W_B\right) = n_{po} \exp\left(\frac{qV_{BC}}{kT}\right) \approx 0$$

Using the quasi-Fermi level description for the current densities:

$$J_n(x) = \mu_n n \frac{\partial E_{Fn}}{\partial x} \qquad J_p(x) = -\mu_p n \frac{\partial E_{Fp}}{\partial x}$$

We will assume that BJT is "decent", so that the hole current in the base is small — small enough that we can ignore it. This implies that $\partial E_{Fp}/\partial x \approx 0$ — the hole quasi-Fermi level is flat.

For non-dengenerate conditions: $n(x)\,p(x) = n_i^2 \exp\left(\frac{E_{Fn} - E_{Fp}}{kT}\right)$

$$\frac{\partial}{\partial x}\left(E_{Fn} - E_{Fp}\right) = kT\left(\frac{n_i^2}{n(x)\,p(x)}\right)\frac{\partial}{\partial x}\left(\frac{n(x)\,p(x)}{n_i^2}\right)$$

Integrate from x to W_B:

$$\left.\frac{n(x)\,p(x)}{n_i^2}\right|_{W_B} - \left.\frac{n(x)\,p(x)}{n_i^2}\right|_{x} = \frac{J_n}{q}\int_x^{W_B} \frac{p(x)}{D_n n_i^2} dy$$

The first term is zero. (Shockly B.C. at reverse-biased collector.) If we evaluate the rest of the expression at $x = 0$:

$$J_n = \frac{1}{\int_0^{W_B} \frac{p(x)}{qD_n n_i^2} dx} \cdot \exp\left(\frac{qV_{BE}}{kT}\right)$$

Apparently, $\left[\int_0^{W_B} \frac{p(x)}{qD_n n_i^2} dx\right]^{-1}$ is the electron current density scale factor, J_{Sn}.

Note, this is not exactly the formulation used by other authors. Often, the electron charge is not included in the integral. In that case, the integral is an "electron flux scale factor".

If the base material is uniform, then n_i^2 is constant.

$$J_{Sn} = q n_i^2 \left[\int_0^{W_B} \frac{p(x)}{D_n} dx \right]^{-1}$$

If there is no high-level injection, the majority hole concentration will be equal to the acceptor doping in the base.

$$J_{Sn} = q n_i^2 \left[\int_0^{W_B} \frac{N_{AB}(x)}{D_n} dx \right]^{-1}$$

If the doping is constant, $p = N_{AB}$, then D_n will also be constant. In that case the integral reduces to $N_{AB} \cdot W_B / D_n$, and the current density factor is

$$J_{Sn} = \frac{q n_i^2 D_n}{N_{AB} W_B}$$

exactly as we had seen for the special case of uniform doping that serving as our launching point.

This integral formulation turns out to be very useful in describing many other non-ideal aspects of BJT operation. It is generally referred to as the Gummel number for the base.

$$\Gamma_B = \int_0^{W_B} \frac{p(x)}{qD_n n_i^2} dx \qquad J_n = \frac{1}{\Gamma_B} \cdot \exp\left(\frac{qV_{BE}}{kT}\right)$$

What's good for the electron current in the npn is is also good for the hole currents:

$$J_{p1} = \frac{1}{\Gamma_E} \cdot \exp\left(\frac{qV_{BE}}{kT}\right) \qquad \Gamma_E = \int_0^{W_E} \frac{n(x)}{qD_p n_i^2} dx$$

$$J_{p2} = \frac{1}{\Gamma_C} \cdot \exp\left(\frac{qV_{BC}}{kT}\right) \qquad \Gamma_C = \int_0^{W_C} \frac{n(x)}{qD_p n_i^2} dx$$

And the two current gain parameters can be expressed in terms of the Gummel numbers for the three regions.

$$\beta_F = \frac{J_{Sn}}{J_{Sp1}} = \frac{\Gamma_E}{\Gamma_B} \qquad \beta_R = \frac{J_{Sn}}{J_{Sp2}} = \frac{\Gamma_C}{\Gamma_B}$$

General base transport factor

In the Gummel number calculation, if we go back one step from the final result (top equation of page 2),

$$-\frac{np}{n_i^2}\bigg|_x = \frac{J_n}{q}\int_x^{W_B} \frac{p(x)}{D_n n_i^2} dy$$

Use the above equation to express the electron concentration $n(x)$:

$$n(x) = -\left(\frac{n_i^2}{p}\right)\left(\frac{J_n}{q}\right)\int_x^{W_B} \frac{p(x)}{D_n n_i^2} dy$$

Integrate over the length of the base

$$\int_0^{W_B} n(x)\, dx = -\left(\frac{J_n}{q}\right)\int_0^{W_B} \left[\frac{n_i^2}{p}\int_x^{W_B} \frac{p(x)}{D_n n_i^2} dy\right] dx$$

$$-\frac{q\int_0^{W_B} n(x)\,dx}{J_n} = \int_0^{W_B}\left[\frac{n_i^2}{p(x)}\int_x^{W_B}\frac{p(x)}{D_n n_i^2}dy\right]dx$$

The numerator on the left-hand side (including the negative sign) is the total electron charge (per unit area) in the base, Q_B. So the expression on the left is the base-transit time.

$$-\frac{q\int_0^{W_B} n(x)\,dx}{J_n} = \frac{Q_{nb}}{J_n} = \tau_B$$

We see that the messy double integral on the right is a generalized expression for the base-transit time and includes any sort of non-uniformity in the base.

$$\tau_B = \int_0^{W_B}\left[\frac{n_i^2}{p(x)}\int_x^{W_B}\frac{p(x)}{D_n n_i^2}dy\right]dx$$

Going back to our simple BJT with uniform regions where $p(x) = N_{AB}$ and D_n and n_i are also constant, we extract the simple result:

$$\tau_B = \frac{W_B^2}{2D_n}$$

Example

A silicon npn BJT has uniformly doped emitter and collector regions ($N_{DE} = 10^{18}$ cm^{-3} and $N_{DC} = 10^{15}$ cm^{-3}). The doping in the base varies exponentially from emitter to collector:

$$N_{AB}(x) = N_{AB}(0)\exp\left(-\frac{x}{L_D}\right) = \left(2.5\times10^{17}\,\text{cm}^{-3}\right)\exp\left(-\frac{x}{0.5\,\mu\text{m}}\right)$$

Assume that the diffusion coefficients can be treated as constants: $D_n = 35$ cm^2/s and $D_p = 10$ cm^2/s. The widths of the 3 three regions are: $W_E = 5$ μm, $W_B = 2$ μm, and $W_C = 10$ μm. Calculate BF and BR.

$$\Gamma_B = \int_0^{W_B}\frac{N_{AB}(x)}{qD_n n_i^2}dx = \frac{N_{AB}(0)}{qD_n n_i^2}\int_0^{2\,\mu\text{m}}\exp\left(-\frac{x}{0.5\,\mu\text{m}}\right)dx = 6.1\times10^{10}\,\frac{\text{cm}^2}{\text{A}}$$

$$J_{Sn} = \frac{1}{\Gamma_B} = 1.64\times10^{-11}\,\frac{\text{A}}{\text{cm}^2}$$

$$J_{Sp1} = \frac{qD_p n_i^2}{N_{DE}W_E} = 1.15\times10^{-13}\,\frac{\text{A}}{\text{cm}^2}$$

$$J_{Sp2} = \frac{qD_p n_i^2}{N_{DC}W_E} = 5.76\times10^{-11}\,\frac{\text{A}}{\text{cm}^2}$$

$$\beta_F = \frac{J_{Sn}}{J_{Sp1}} = 143$$

$$\beta_F = \frac{J_{Sn}}{J_{Sp1}} = 0.285$$