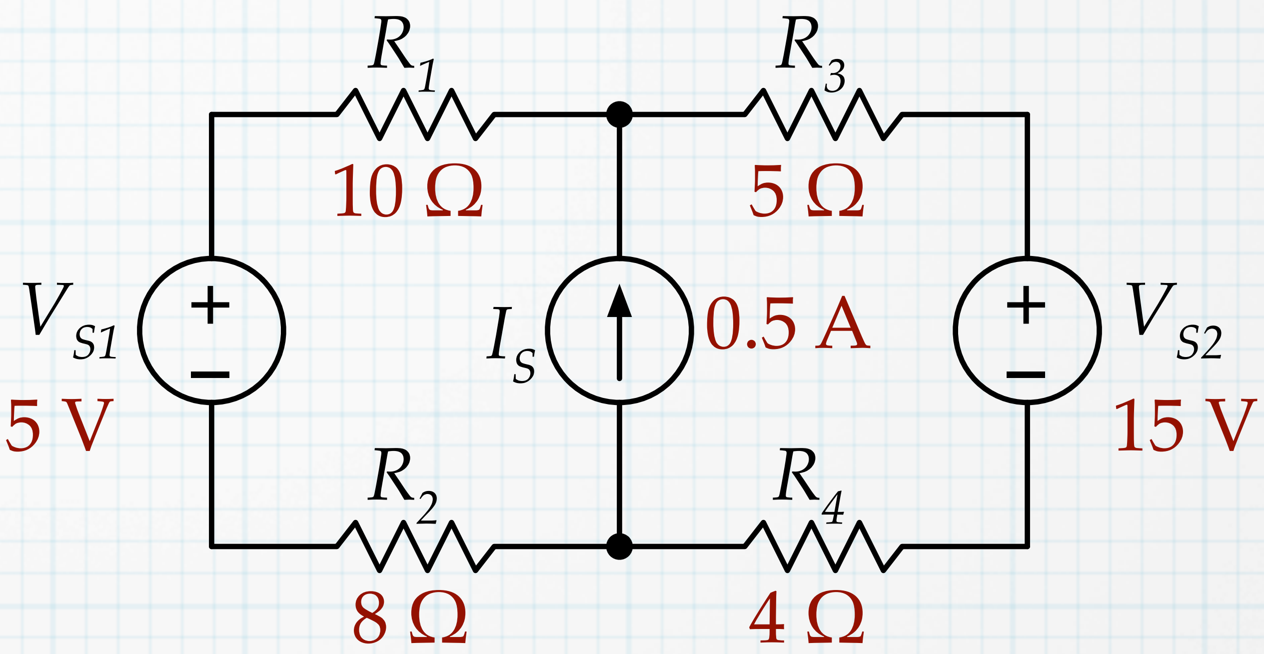
A trick mesh-current situation

When using the node-voltage method, we usually try to choose the ground connection so that it is at one side of a voltage. We also saw that if a voltage source was not connected to ground, then we needed to modify the node-voltage approach to account for the current through the "ungrounded" source. We needed to use an "auxiliary" equation account for the unknown source current or we needed to use the "supernode" method to avoid the source current altogether.

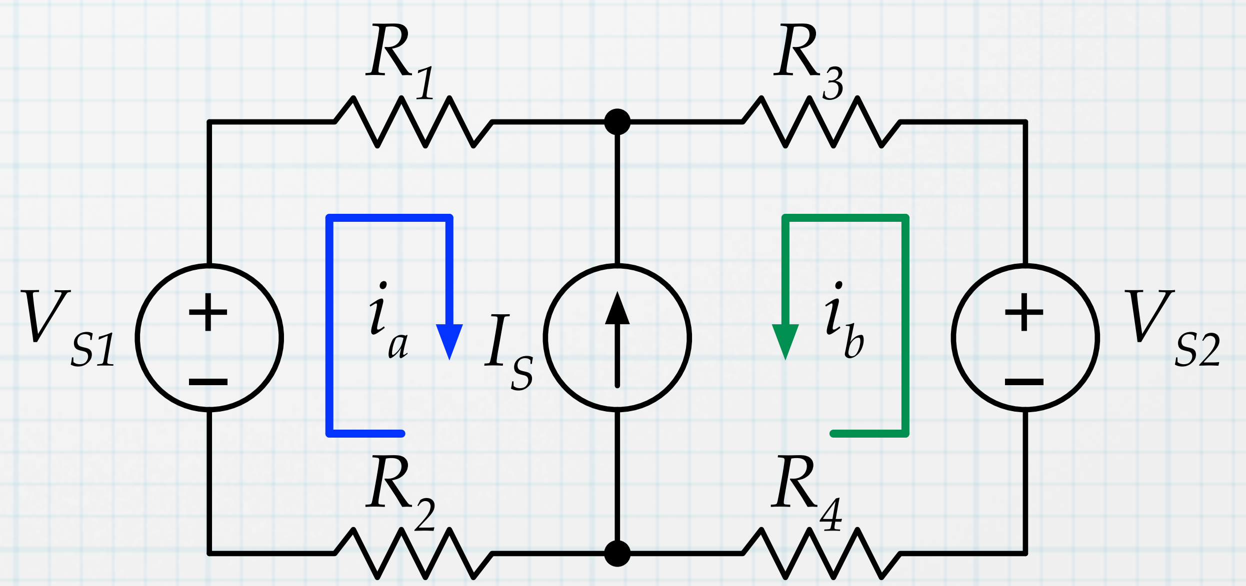
A similar problem crops up in the mesh-current method when a current source is located in two meshes simultaneously, i.e. it is not in an outside branch of a mesh. In that case, the voltage across the current source would show up on the KVL equations. The voltage across the current source becomes another unknown in the set of mesh-current equations.

Example

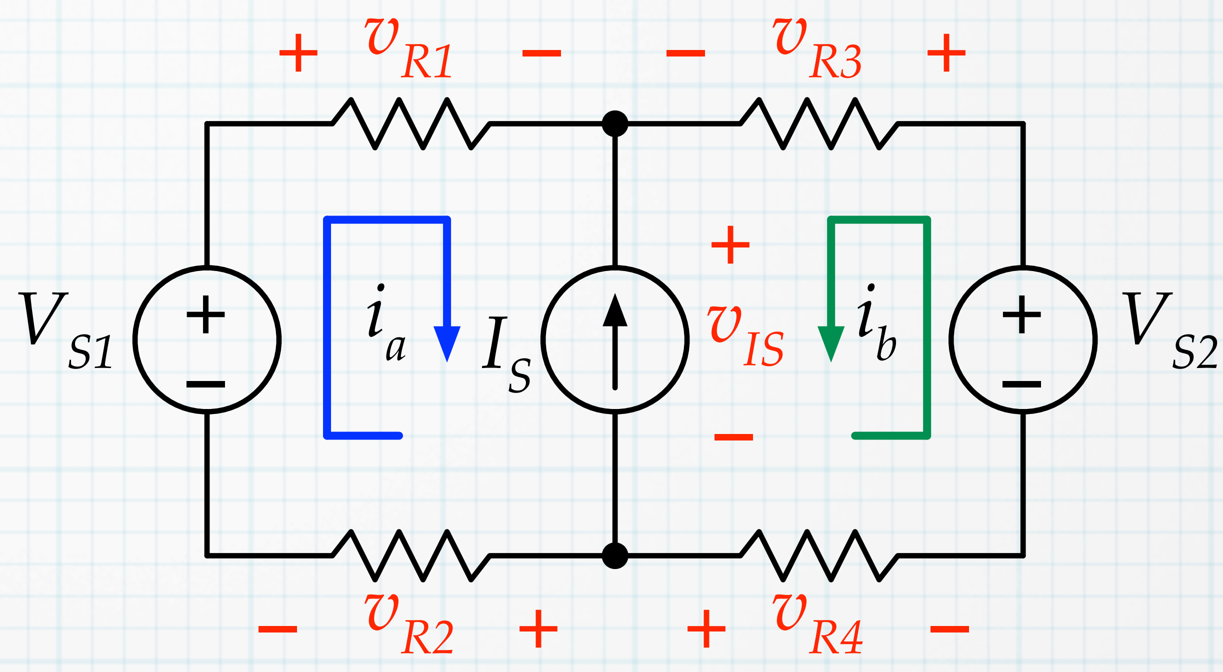
To see the problem and how to handle it, consider the circuit at right.



There are clearly two meshes and so we define two mesh currents.



Define the voltages for the components around each mesh and write KVL equations.



a: $V_{S1} - v_{R1} - v_{IS} - v_{R2} = 0.$

b: $V_{S2} - v_{R3} - v_{IS} - v_{R4} = 0.$

Use Ohm's law to write the resistor voltages in terms of currents and then write the mesh-current equations.

$v_{R1} = R_1 i_a \qquad v_{R2} = R_2 i_a \qquad v_{R3} = R_3 i_b \qquad v_{R4} = R_4 i_b$

a: $V_{S1} - R_1 i_a - v_{IS} - R_2 i_a = 0.$

b: $V_{S2} - R_3 i_b - v_{IS} - R_4 i_b = 0.$

Now we see the problem. The voltage across the current source gives us a third unknown.

Find an auxiliary equation

a: $V_{S1} - R_1\, i_a - v_{IS} - R_2 i_a = 0.$

b: $V_{S2} - R_3\, i_b - v_{IS} - R_4\, i_b = 0.$

Having three unknowns but only two equations suggests that we need a third equation. The current source provides the third (auxiliary) equation. In looking at the circuit, we see that I_S is related to i_a and i_b:

$I_S = -(i_a + i_b)$

Using this third equation together with the mesh equations, we can use whatever algebraic gymnastics we like to solve the equations. One approach is use the first two equations to eliminate v_{IS} and then use the resulting equation together with the auxiliary equation to solve for i_b and i_b.

a: $V_{S1} - R_1\, i_a - v_{IS} - R_2 i_a = 0. \longrightarrow v_{IS} = V_{S1} - R_1\, i_a - R_2 i_a$

b: $V_{S2} - R_3\, i_b - (V_{S1} - R_1\, i_a - R_2 i_a) - R_4\, i_b = 0.$

$$-(R_1 + R_2)\, i_a + (R_3 + R_4)\, i_b = V_{S2} - V_{S1}$$ (equation b re-arranged)

$$i_a + i_b = -I_S$$ (auxiliary equation)

$$-(R_1 + R_2)\, i_a + (R_3 + R_4)(-I_S - i_a) = V_{S2} - V_{S1}$$

$$-(R_1 + R_2 + R_3 + R_4)\, i_a = V_{S2} - V_{S1} + (R_3 + R_4)(I_S)$$

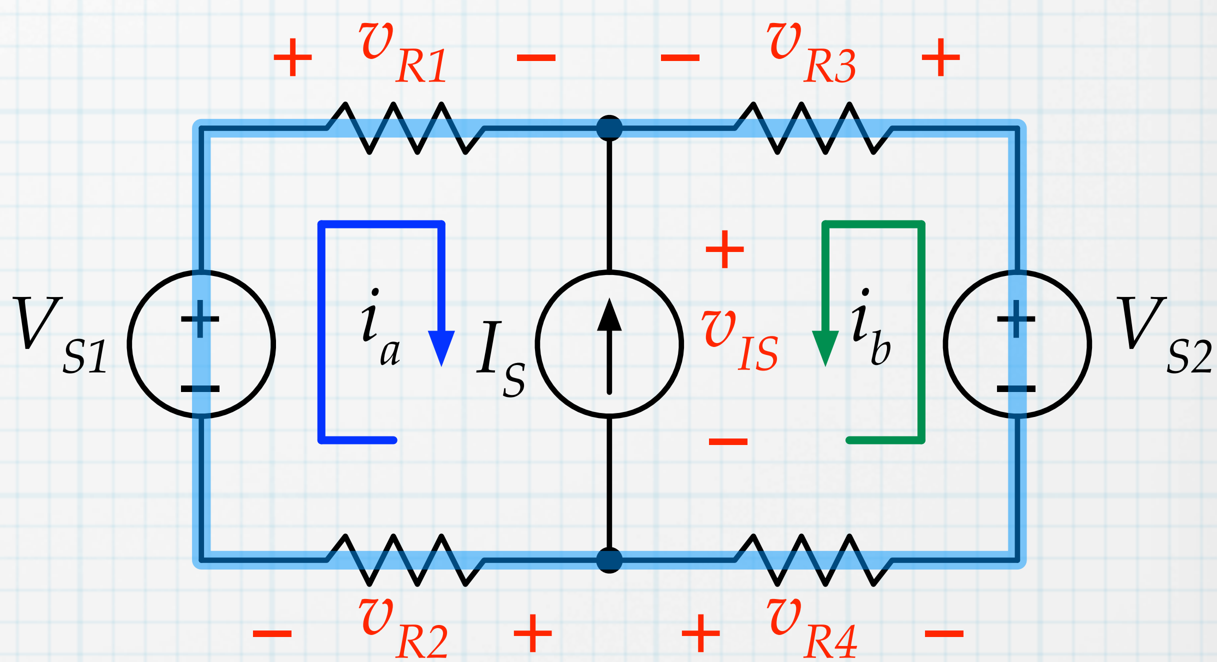
$$i_a = -\frac{V_{S2} - V_{S1} + (R_3 + R_4)\, I_S}{R_1 + R_2 + R_3 + R_3}$$

$$= -\frac{15\text{ V} - 5\text{ V} + (5\Omega + 4\Omega)\,(0.5\text{ A})}{10\Omega + 8\Omega + 5\Omega + 4\Omega} = -0.537\text{ A}$$

and then $i_b = +0.037$ A

Use a "super mesh"

An alternative approach is to use a larger mesh that does not include the interior current source. The basic idea is go clear around the current source and avoid v_{IS} entirely. In this example, the only other mesh is the big one that includes the two voltage sources, as indicated by the blue highlight below. Using KVL around the super mesh gives



$$V_{S1} - v_{R1} + v_{R3} - V_{S2} + v_{R4} - v_{R2} = 0.$$

Insert the currents.

$$V_{S1} - R_1 i_a + R_3 i_b - V_{S2} + R_3 i_b - R_2 i_a = 0.$$

Use the relationship between the current source and the two mesh currents.

$$i_a + i_b = -I_S$$

These are the exact same equations that we solved on the previous page.