RL transients

Circuits having inductors:

- At DC – inductor is a short circuit, just another piece of wire.
- Transient – a circuit changes from one DC configuration to another DC configuration (a source value changes or a switch flips). There will be a transient interval while the voltages and currents in the inductors change.
- AC – currents and voltages are changing continuously, so inductors are "amping up" and "de-amping" continuously. This requires special techniques and is the next topic for EE 201.

Solving a circuit with transient changes

1. Determine the DC currents in the inductors before the change occurs. These may be given, or you may have to solve for them from the original configuration.

2. Let the change occur instantaneously at time $t = 0$. The inductors will maintain their currents into the "instant" just after the change. (Recall: inductor current cannot change instantaneously.)

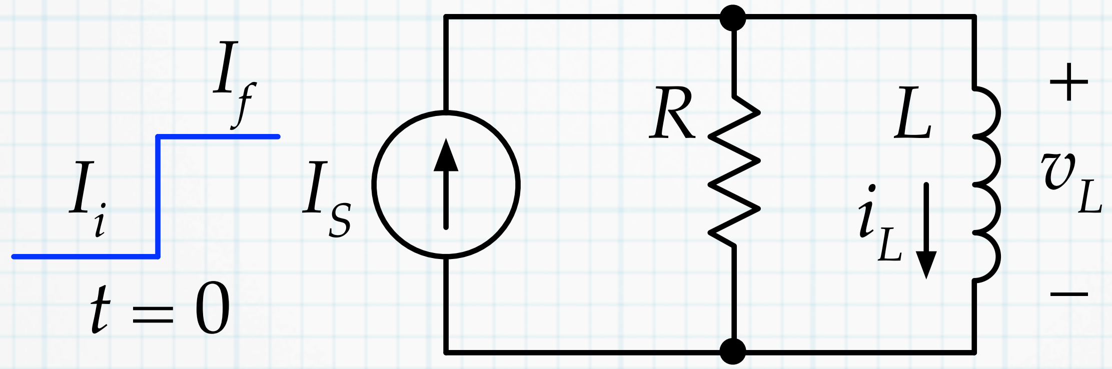
3. Analyze the circuit. Since inductor voltage depend on di_L/dt, the result will be a differential equation.

4. Solve the differential equation, using the inductor currents from before the change as the initial conditions.

5. The resulting equation will describe the "amping" (or "de-amping") of the inductor current during the transient and give the final DC value once the transient is complete.

In practice, we will solve only one circuit and try to understand it completely. Then, when we encounter other circuit configurations, we will make those fit the prototype.

Simple RL circuit transient (physics)



In the circuit, I_S abruptly changes value from I_i to I_f at $t = 0$. Assume that the source was at I_i for a "very long time" before t = 0.

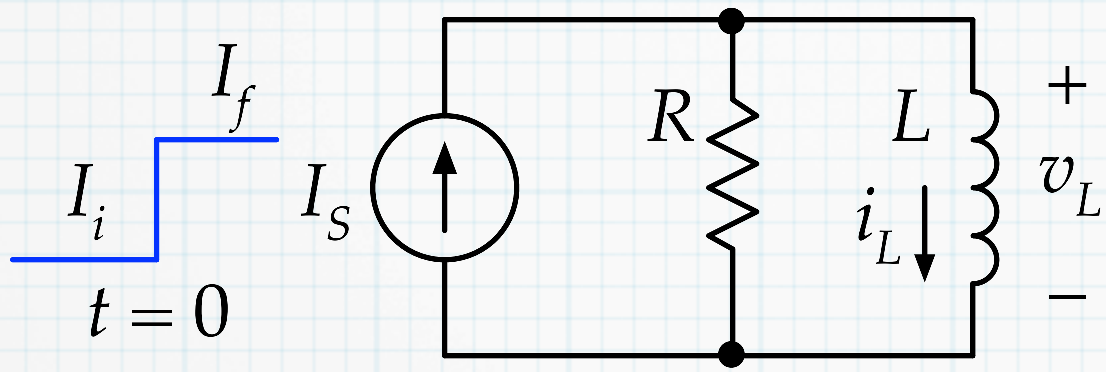
1. For $t < 0$, $i_L = I_i$ and $v_L = 0$. Therefore, $i_R = 0$.
2. At $t = 0$, I_S changes. But $i_L = I_i$ (still), and so any excess current must flow through the resistor. Consequently, v_L jumps abruptly:

$$v_L(0) = R\left[I_f - i_L(0)\right] = R\left[I_f - I_i\right]$$

3. For $t > 0$, i_L increases. As i_L increases, i_R and v_L will decrease.

4. After a "sufficiently long time", i_L will "amp up" to I_f. The current through the resistor and voltage across it both drop to zero. The transient is complete.

Simple RL circuit transient (math)



In the circuit, I_S abruptly changes value from I_i to I_f at t = 0. Assume that the source was at I_i for a "very long time" before t = 0.

For $t < 0$, $i_L = I_i$ and $v_L = 0$.

$$v_L = v_R$$

$$L\frac{di_L}{dt} = R\left(I_f - i_L\right)$$

$$\frac{di_L}{I_f - i_L} = \frac{dt}{{}^L/_R}$$

$$\int \frac{di_L}{I_f - i_L} = \int \frac{dt}{{}^L/_R}$$

$$\int_{I_i}^{i_L(t)} \frac{di_L}{I_f - i_L} = \frac{1}{{}^L/_R}\int_0^t dt'$$

$$-\ln\left(I_f - i_L\right)\Big|_{I_i}^{i_L(t)} = \frac{t}{{}^L/_R}$$

$$\ln\left[\frac{I_f - I_i}{I_f - i_L(t)}\right] = \frac{t}{{}^L/_R}$$

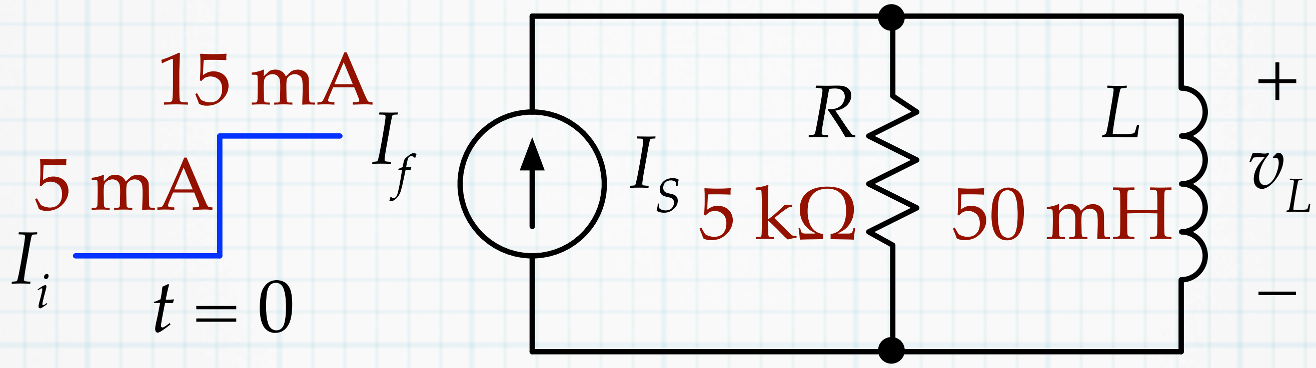
$$i_L(t) = I_f - \left[I_f - I_i\right]\exp\left(-\frac{t}{{}^L/_R}\right)$$

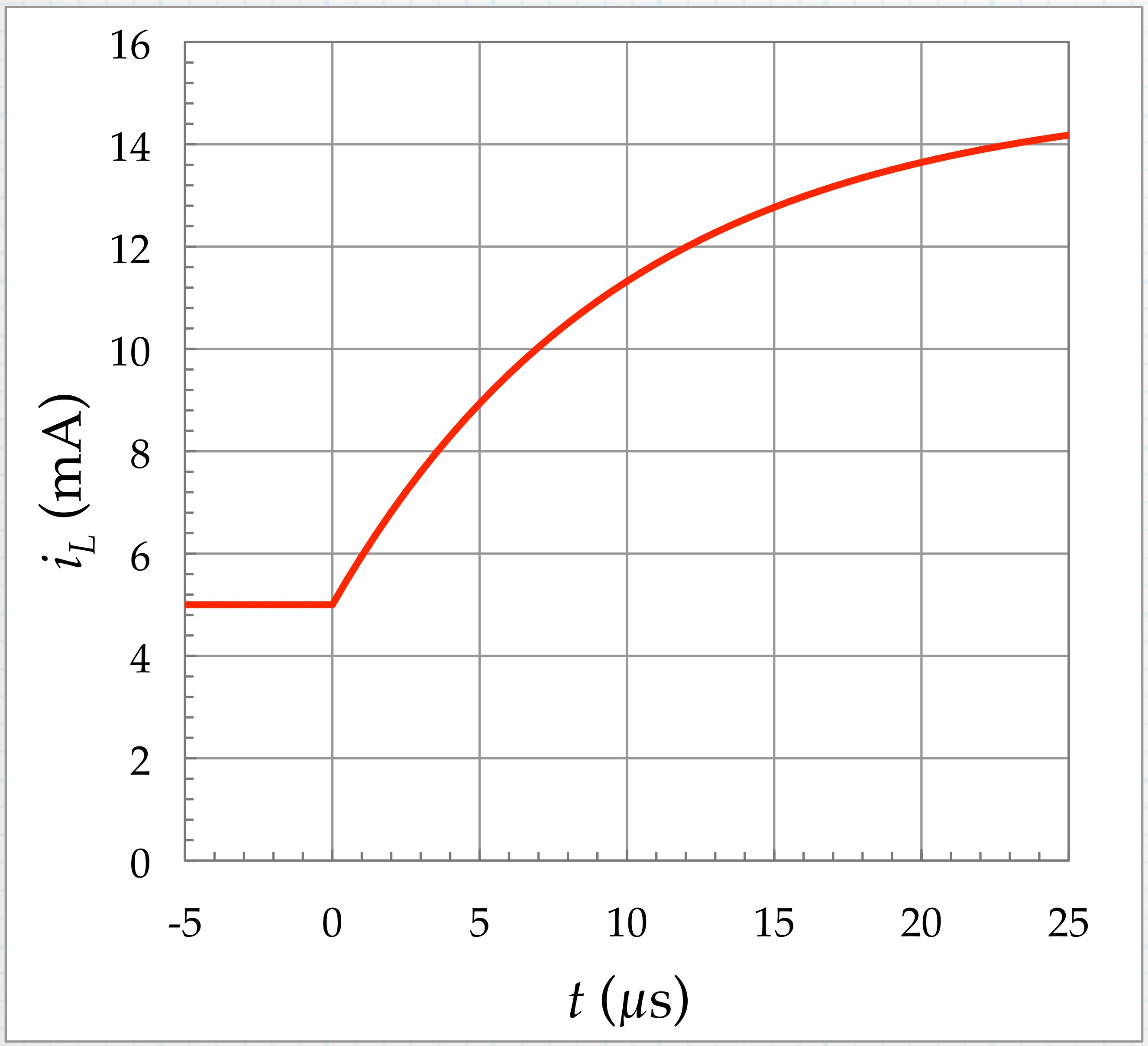
$$i_L(t) = I_f - \left[I_f - I_i\right] \exp\left(-\frac{t}{L/R}\right)$$

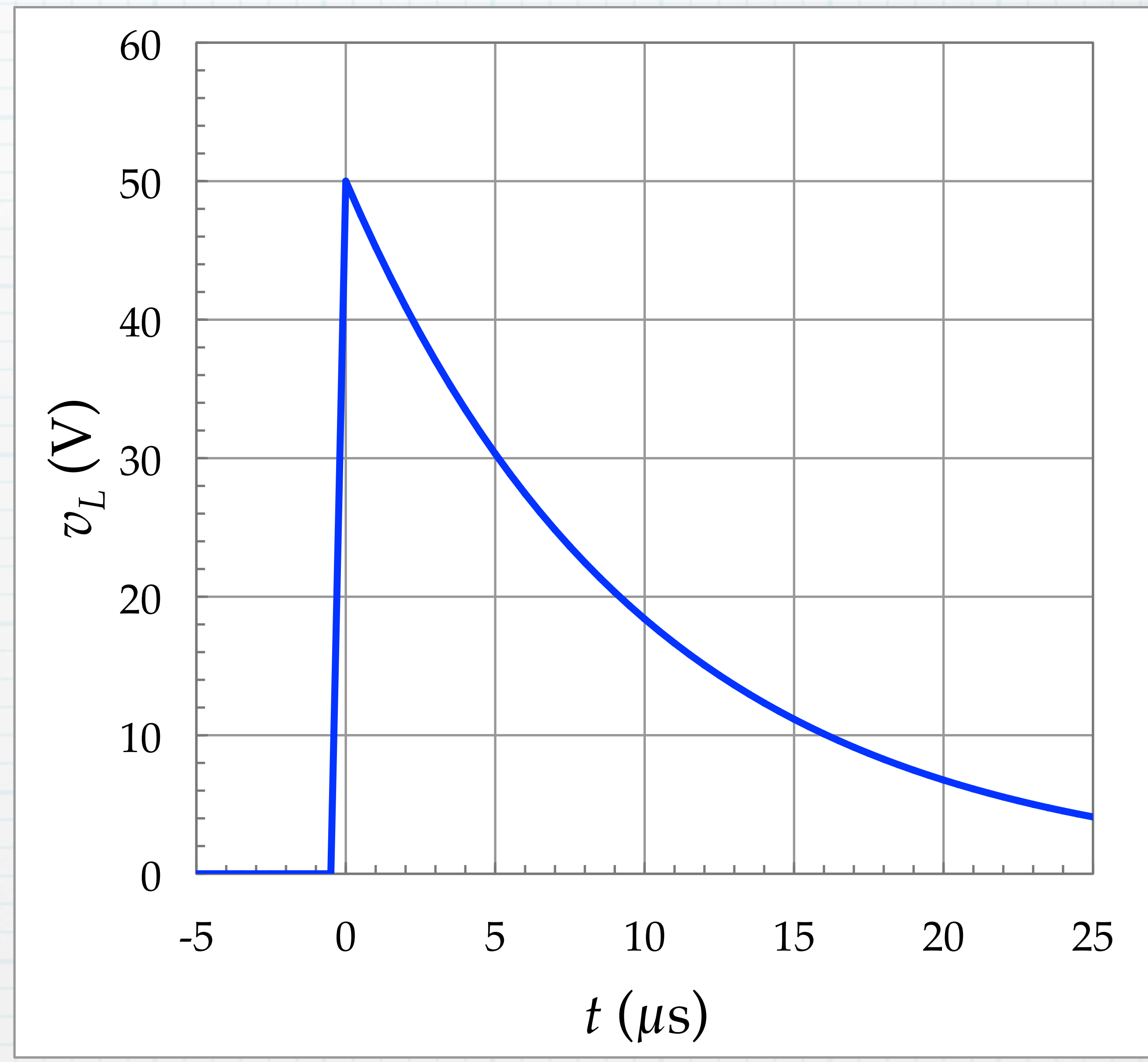
- At $t = 0$, $i_L = I_i$, as expected.
- As $t \to \infty$, $i_L \to I_f$, also as expected.
- In the between the inductor current changes according to a decaying exponential, given above.
- At $t = 0$, v_L jumps to up to a maximum value.
- For $t > 0$, v_L decays away exponentially, as the current approaches its final value.

$$v_L(t) = \left(I_f - i_L\right) R \qquad v_L(t) = L\frac{di_L}{dt} = R\left(I_f - I_i\right) \exp\left(-\frac{t}{L/R}\right)$$

Finally, note that the equation works just as well for "amping down" as it does for "amping up". In the example, we implied $I_f > I_i$, but that was never a requirement in the derivation of the equation.



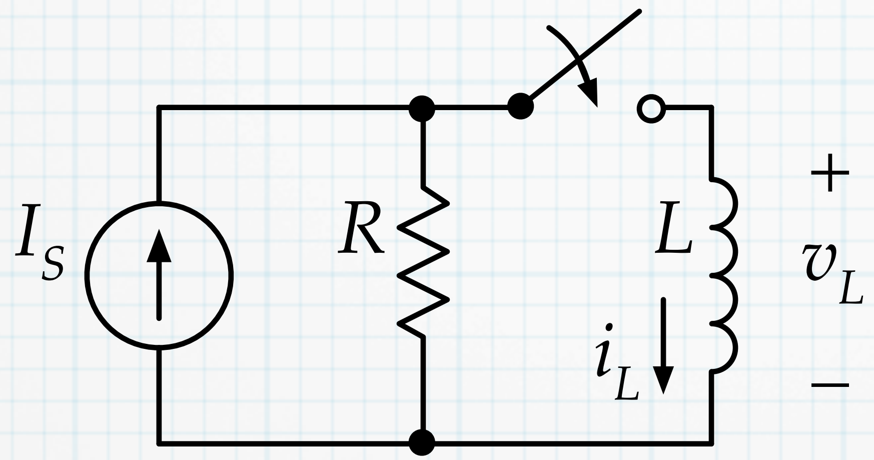




Plots of capacitor voltage and current for a simple *RL* circuit with I_f = 15 mA, V_i = 5 mA, R = 5 kΩ, L = 50 mH (L/R = 50 μs).

Using a switch

The same transient phenomena occurs when using a switch to change a circuit.



1. For $t < 0$, the inductor may have a current flowing (depending on the circuit configuration), $i_L(t < 0) = I_i$. Also, $v_L = 0$.

2. At $t = 0$, the switch closes. The inductor maintains its current, $i_L(0) = I_i$. The inductor voltage jumps abruptly to

$$v_L\left(0\right) = R\left[I_f - i_L\left(0\right)\right] = R\left[I_f - I_i\right]$$

3. For $t > 0$, the inductor current rises. The voltage drops as the current rises.

$$i_L\left(t\right) = I_f - \left[I_f - I_i\right]\exp\left(-\frac{t}{{}^{L}/_{R}}\right)$$

RL time constant

- The decay is characterized by a time constant, $\tau = L/R$.
  (Check the units.)

- L/R determines the time scale for the transient.

  If $R = 1\text{k}\Omega$ and $L = 1$ H, $\tau = L/R = 1$ ms.

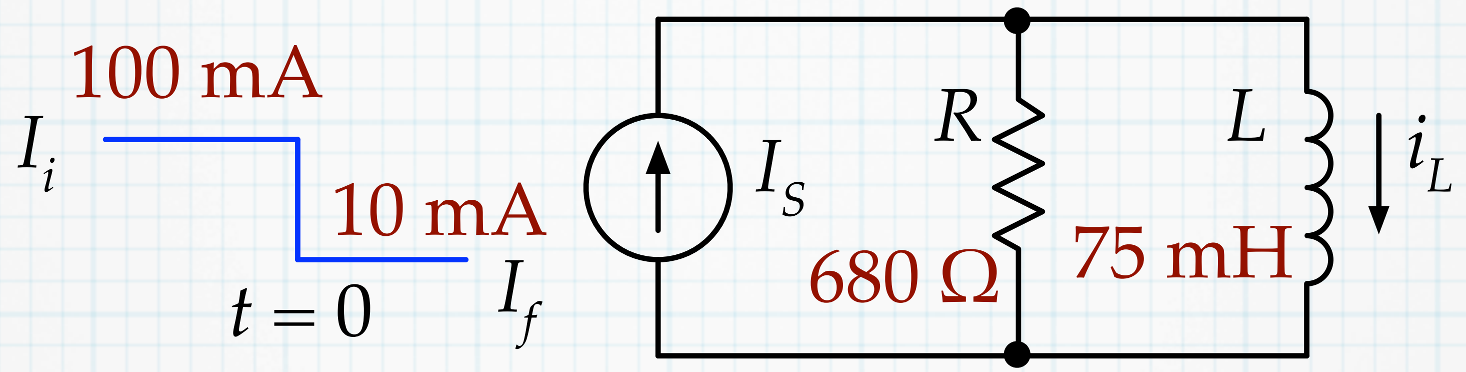
  If $R = 10$ kΩ and $L = 10$ mH, $\tau = L/R = 1\ \mu$s.

- After about 5 time constants, most of the changed has occurred.

  If $t = 5(L/R)$ → exp(–5) = 0.0067 → 99.3% of the transition is done.

  (Theoretically, i_L never really gets to I_f. But as engineers, we have to be practical.)

Example



In the *RL* circuit above, I_S abruptly changes value from I_i = 100 mA to I_f = 10 mA at t = 0. Assume that the source was at I_i for a "very long time" before t = 0. Find the expression for the inductor current as a function of time. Find the time at which i_L = 50 mA.

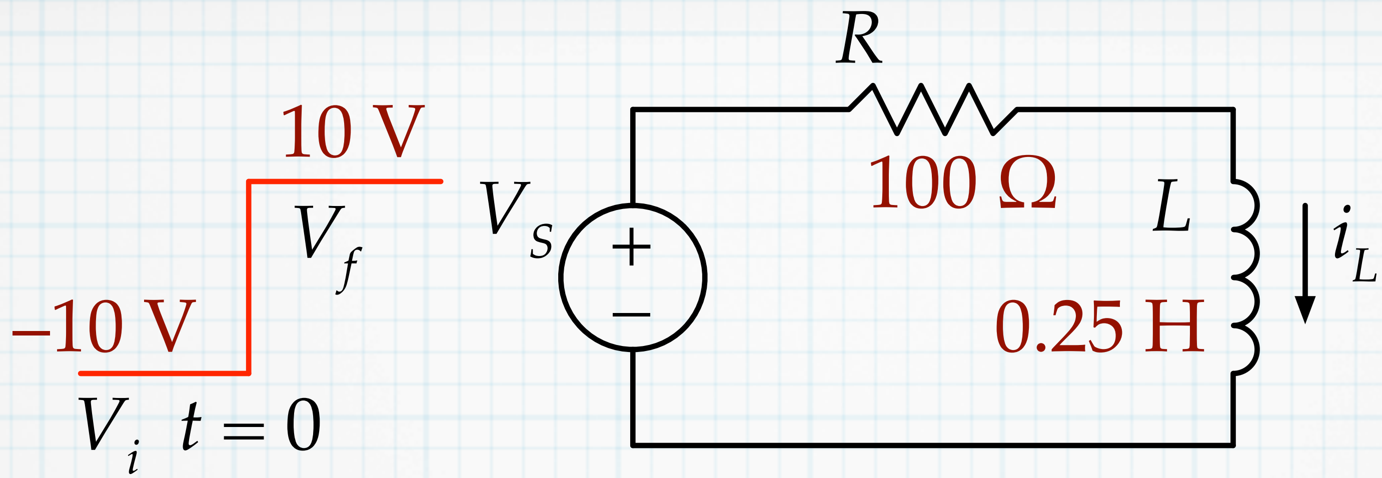
Apply the transient function directly, with I_i = 100 mA, I_F = 10 mA, and L/R = 75 mH/680 Ω = 110 μs.

$$i_L(t) = I_f - [I_f - I_i)] \exp\left(-\frac{t}{L/R}\right) = [10\text{mA}] + [90\text{mA}] \exp\left(-\frac{t}{110\mu\text{s}}\right)$$

$$50\text{mA} = [10\text{mA}] + [90\text{mA}] \exp\left(-\frac{t'}{110\mu\text{s}}\right)$$

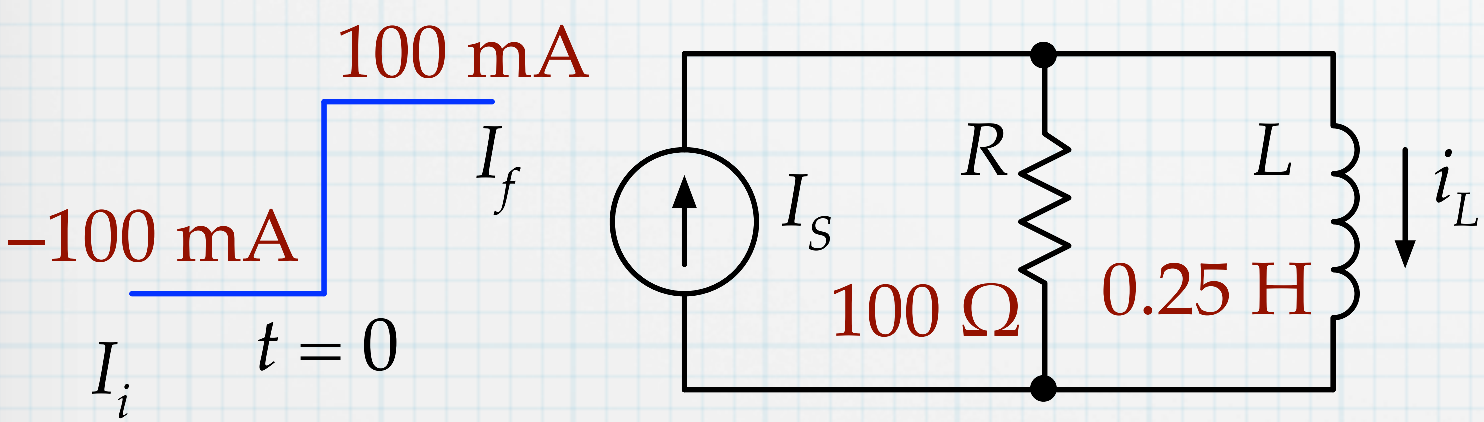
$$t' = (110\mu\text{s}) \ln\left[\frac{90\text{mA}}{40\text{mA}}\right] = 89\mu\text{s}$$

Example 2



The voltage source in the above circuit abruptly changes from – 10 V to +10 V at t = 0. Find expressions for the inductor current and inductor voltage. Find the time at which the inductor current crosses 0 mA.

The circuit is in the wrong form to the equation directly. Instead use a source transformation to change it to the "standard" form.



$$i_L(t) = I_f - \left[I_f - I_i)\right] \exp\left(-\frac{t}{L/R}\right)$$

$$= [0.1\text{A}] - [0.2\text{A}] \exp\left(-\frac{t}{2.5\text{ms}}\right)$$

$$v_L(t) = L\frac{di_L}{dt}$$

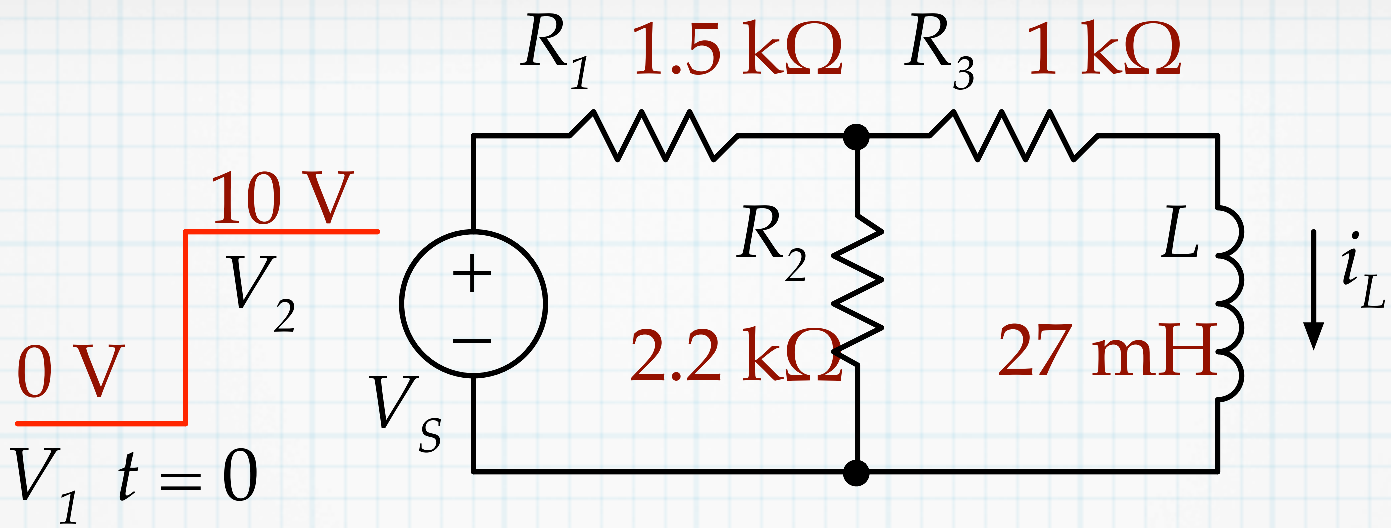
$$= R\left[I_f - I_i\right] \exp\left(-\frac{t}{L/R}\right)$$

$$= [10\text{V}] \exp\left(-\frac{t}{2.5\text{ms}}\right)$$

$$0 = [100\text{mA}] - [200\text{mA}] \exp\left(-\frac{t'}{2.5\text{ms}}\right)$$

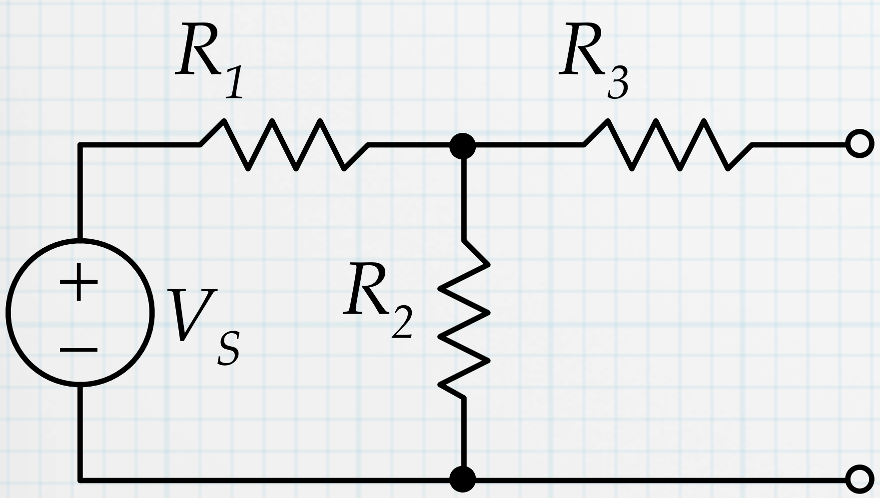
$$t' = (2.5\text{ms}) \ln\left[\frac{200\text{mA}}{100\text{mA}}\right] = 1.73\text{ms}$$

Example 3



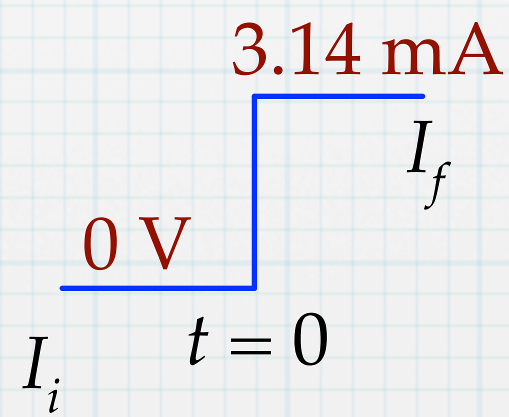
In the *RL* circuit above, V_S abruptly changes value from V_1 to V_2 at t = 0. Find the expression for the inductor current as a function of time.

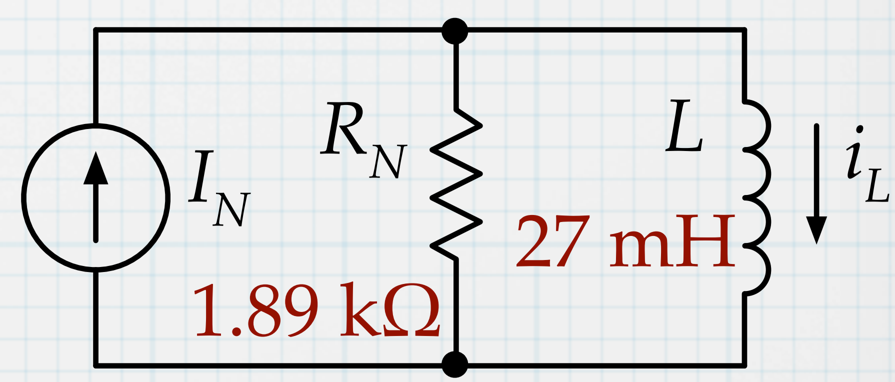
Find the *Norton* equivalent for the circuit attached to the inductor.



$$I_N = \frac{V_S}{R_1 + \left(1 + \frac{R_1}{R_2}\right) R_3} = 0.314\ V_S.$$

$$R_N = R_3 + R_1 \| R_2 = 1.89\ \text{k}\Omega.$$





$t < 0$, $I_i = 0$.

$t \geq 0$, $I_f = 3.14$ mA.

$L/R_N = (27\ \text{mH})/(1.89\ \Omega) = 14.3\ \mu\text{s}$

$$i_L(t) = [3.14\text{mA}]\left[1 - \exp\left(-\frac{t}{14.3\mu\text{s}}\right)\right]$$

To study:

1. Work through the solution to the differential equation (slide 4) and make sure that you understand it thoroughly.
2. Try to find the solution to Example 3 *without* using the Thevenin equivalent. It can be done, although it might be a bit messy.
3. For the example shown on slide 6, calculate the inductor energy before and after the transient. Calculate the total energy delivered by the source during the transient. Show that everything balances. (Don't forget about the power consumed in the resistor.)