

The mesh-current method

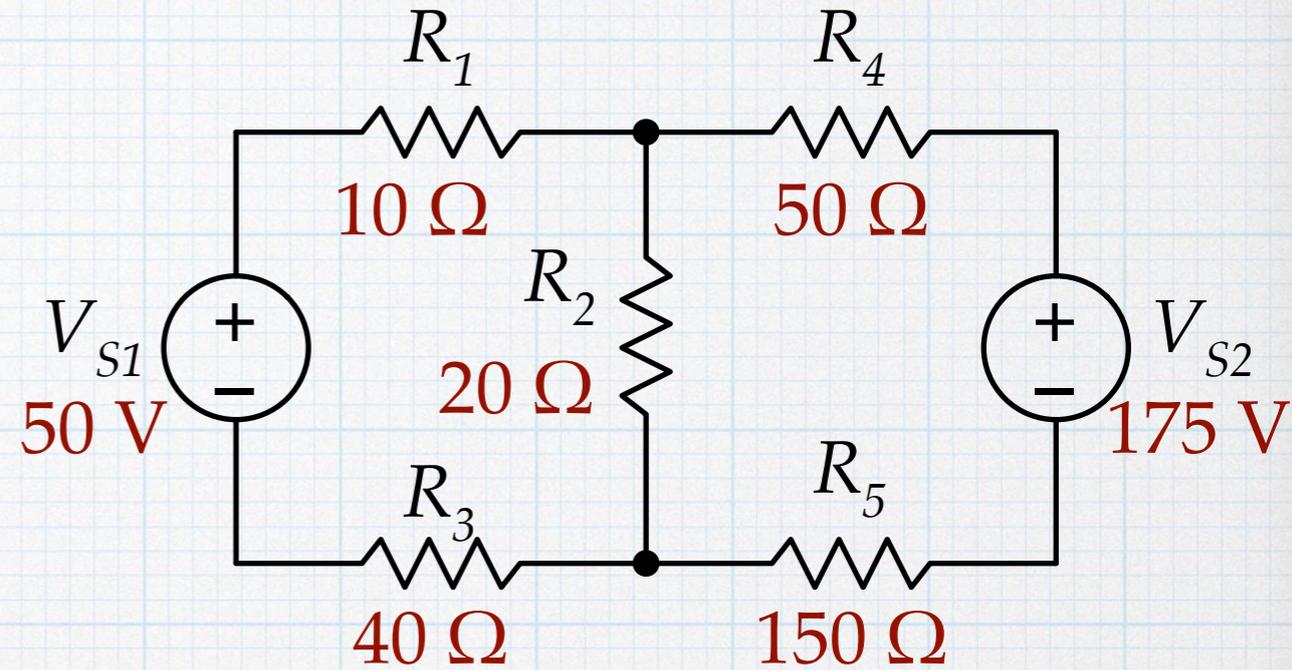
- Equivalent resistance
- Voltage / current dividers
- Source transformations
- Node voltages
- **Mesh currents**
- Superposition

Mirror image of the node-voltage method.

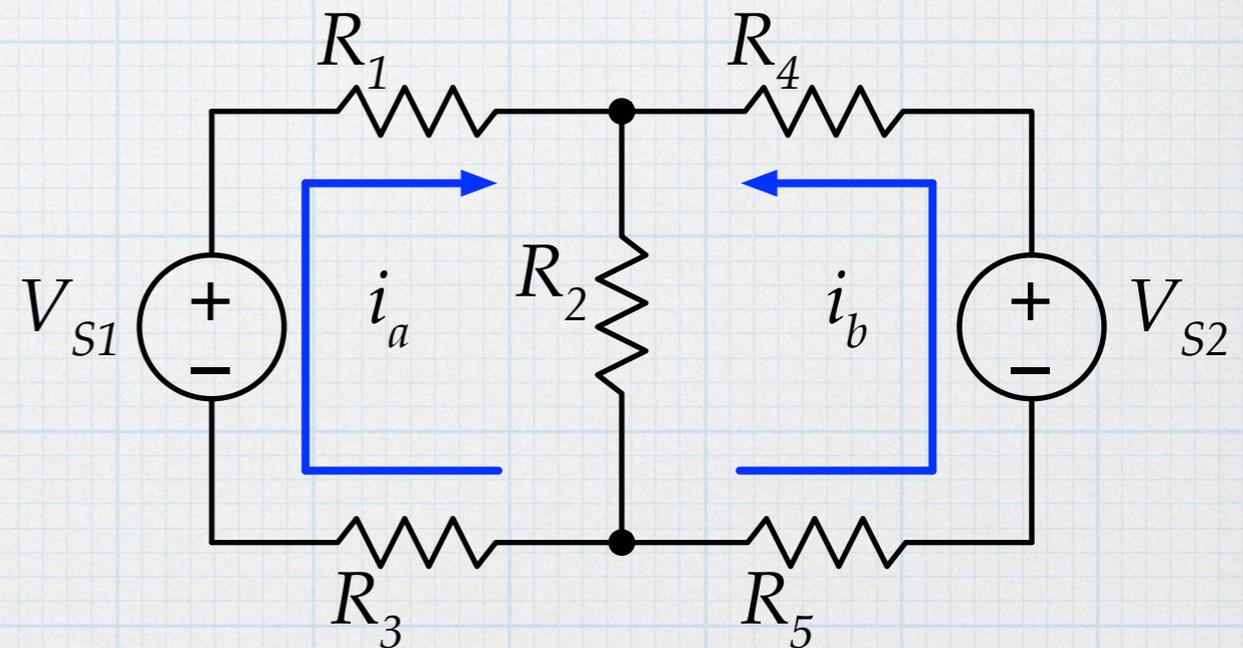
- Define mesh currents flowing around the loops that make up a circuit.
- Then use KVL to relate the voltages around each loop.
- Convert voltage equations to mesh-current equations using Ohm's law.

An example

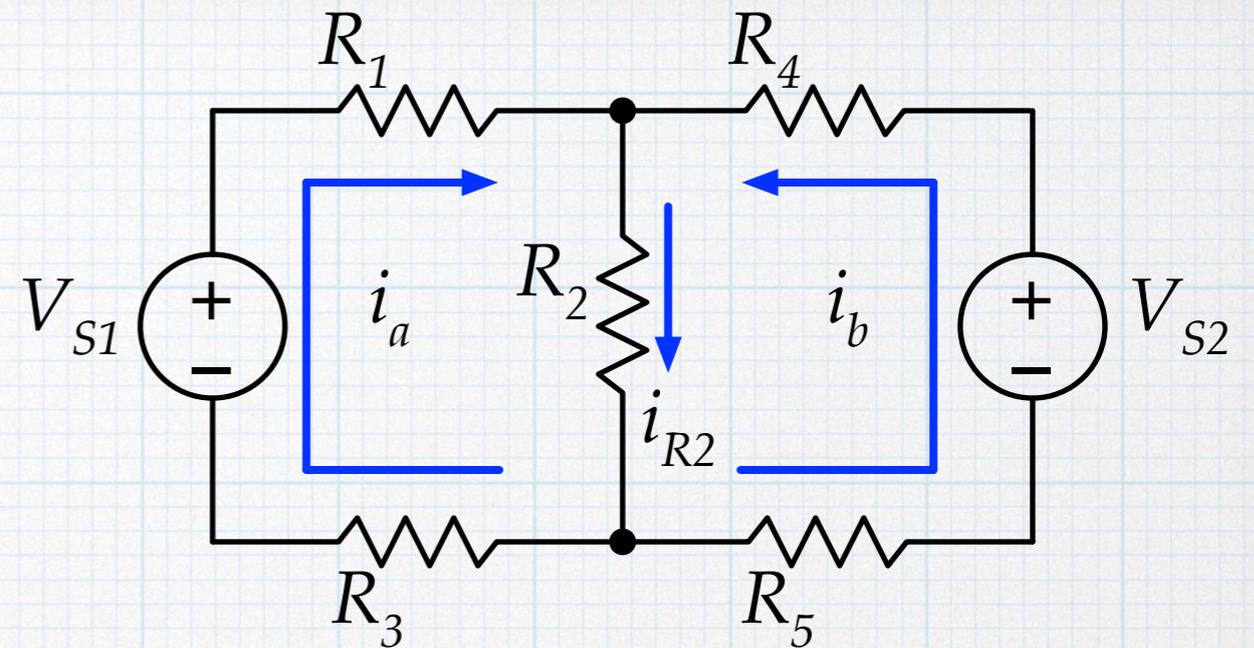
Let's re-consider the last circuit that we solved with the node-voltage method, where we wanted to find v_{R_2} in the circuit at right. It appeared to be a simple circuit, but it was difficult because of all the nodes and the need to use an auxiliary equation.



Rather than focusing on nodes, let's consider the currents around the outside branches. We note that R_1 , V_{S1} , and R_3 are in series and will carry the same current, which we can denote as i_a . Similarly, R_4 , V_{S2} , and R_5 are in series, and we can denote their common current as i_b . Again, the current directions we choose here at the outset is arbitrary.



Furthermore, using KCL at the top center node, we see that $i_{R2} = i_a + i_b$.

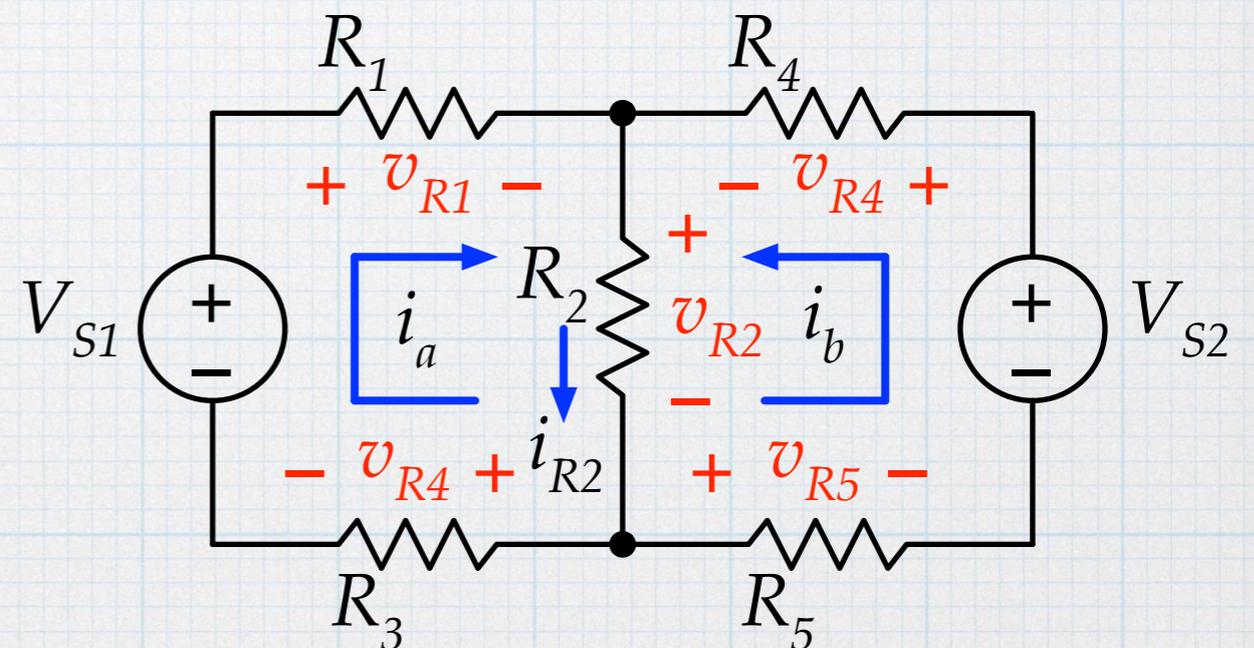


Denoting the voltages on each component, we can use KVL around each of the loops. On the left:

$$V_{S1} - v_{R1} - v_{R2} - v_{R3} = 0$$

And on the right:

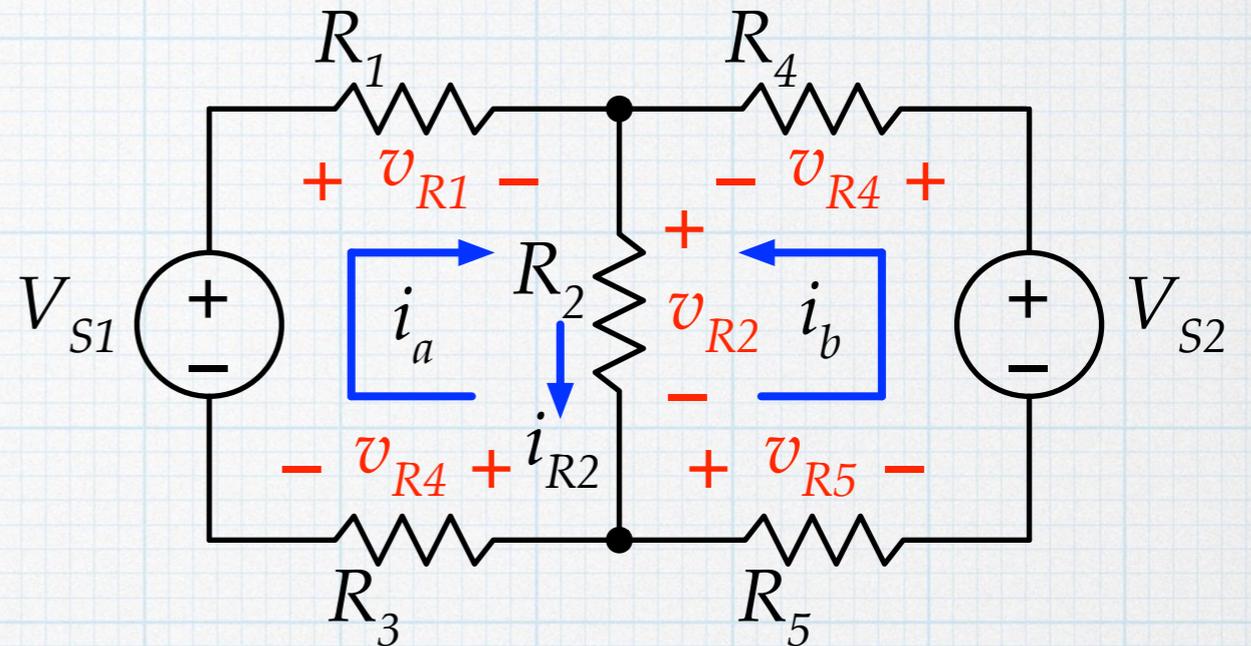
$$V_{S2} - v_{R4} - v_{R2} - v_{R5} = 0$$



We can write the resistor voltages in the two equations in terms of the currents, using Ohm's law. Recall that $i_{R2} = i_a + i_b$

$$V_{S1} - i_a R_1 - (i_a + i_b) R_2 - i_a R_3 = 0$$

$$V_{S2} - i_b R_4 - (i_a + i_b) R_2 - i_b R_5 = 0$$



Wait...what!? The result is two equations in the two unknown currents, i_a and i_b . This is much easier than the mess we had when using node-voltages. What we have done here outlines the *mesh-current* method. Let's finish the problem and then re-examine the basic approach.

Re-writing the equations:

$$(R_1 + R_2 + R_3)i_a + R_2 i_b = V_{S1}$$

$$(70 \Omega)i_a - (20 \Omega)i_b = 50 \text{ V}$$

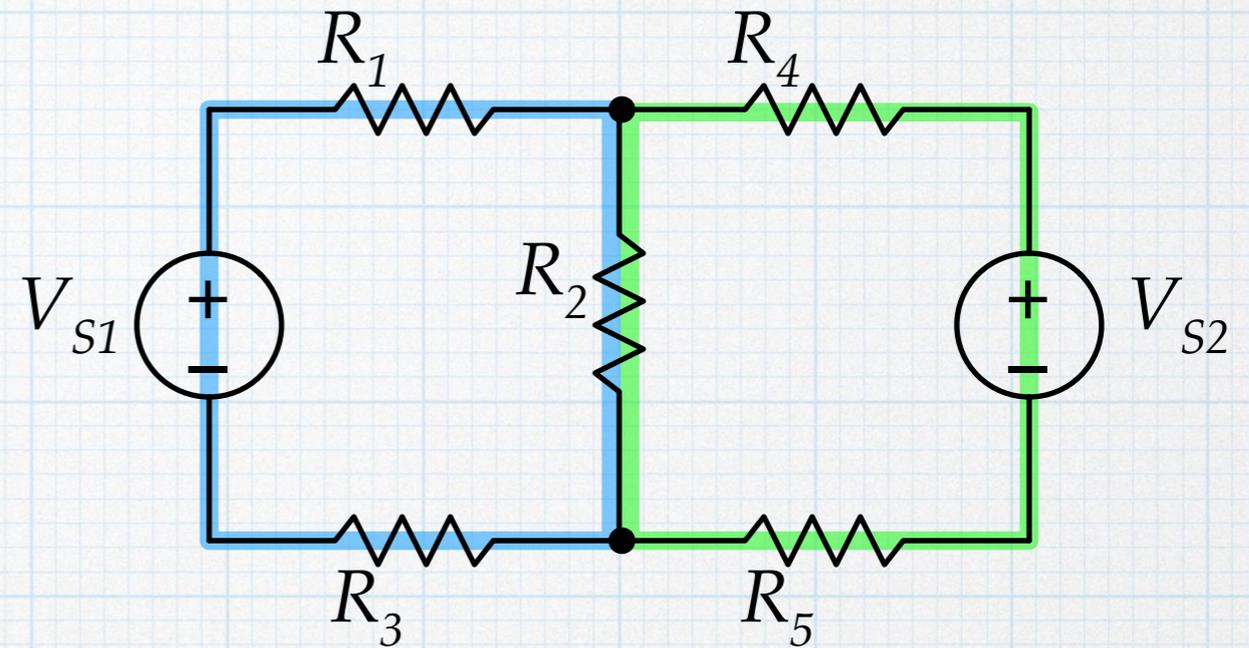
$$R_2 i_a + (R_1 + R_2 + R_3)i_b = V_{S2}$$

$$-(20 \Omega)i_a + (120 \Omega)i_b = 100 \text{ V}$$

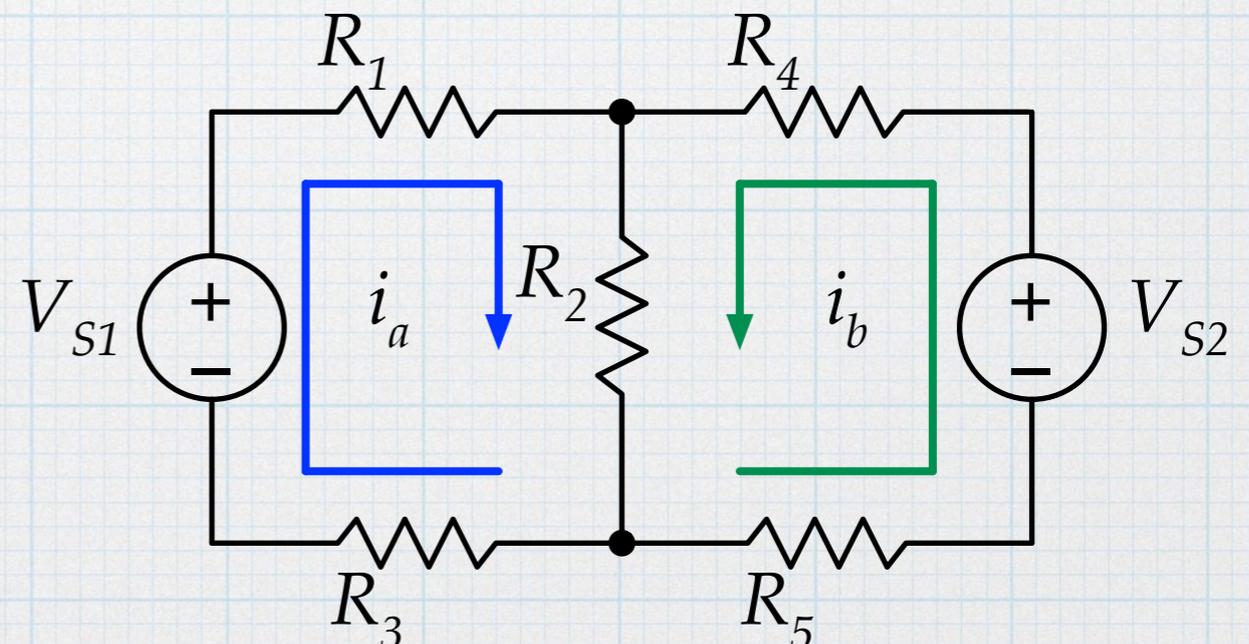
Solving give: $i_a = 0.5 \text{ A}$, $i_b = 0.75 \text{ A}$, and $v_{R2} = (20 \Omega)(0.5 \text{ A} + 0.75 \text{ A}) = 25 \text{ V}$.

The mesh-current method

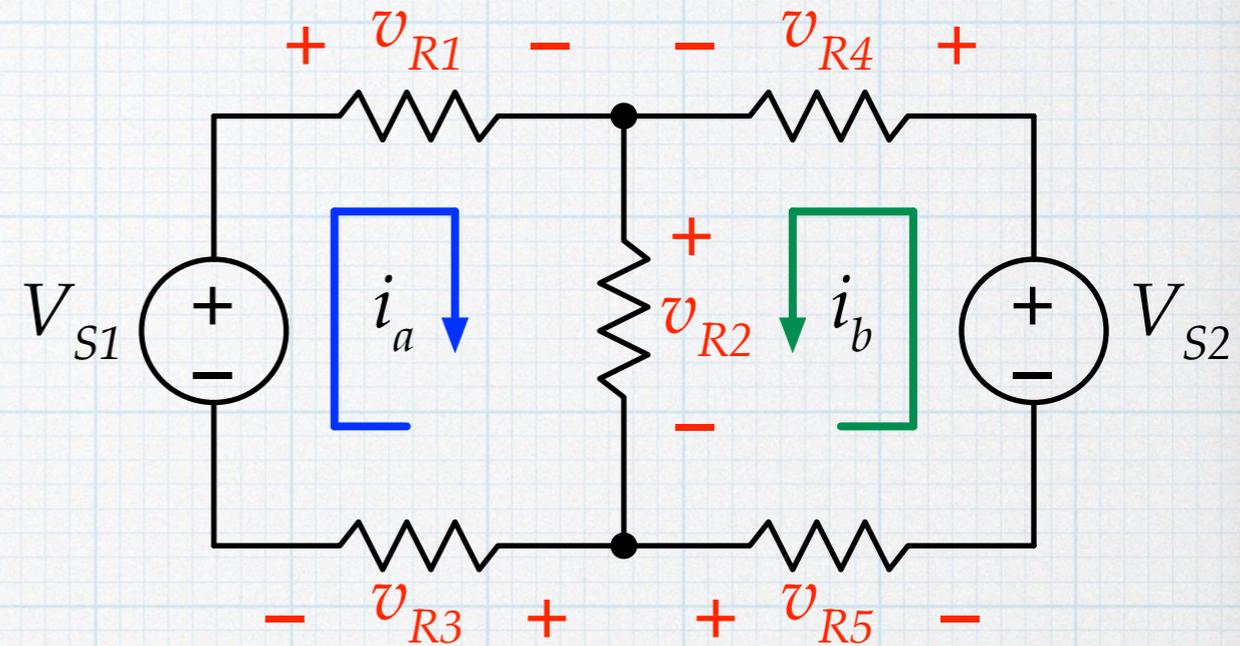
1. The mesh current approach starts by identifying the meshes (or loops) that make up the circuit. Generally, we want the set of the smallest meshes that completely define the circuit. In this case, there are 2.



2. Each mesh will have a *mesh current* (or *loop current*) that circulates around the loop. The currents in branches that are shared by two meshes will be some combination of the the meshes, according to KCL.



3. Identify the voltage drops on each component. Make sure that the resistor polarities are commensurate with the directions of the mesh currents. (Initially, it is a good idea to label all the voltage polarities explicitly.)



4. Write KVL equations around each mesh.

$$V_{S1} - v_{R1} - v_{R2} - v_{R3} = 0$$

$$V_{S2} - v_{R4} - v_{R2} - v_{R5} = 0$$

5. Use Ohm's law to write each resistor voltage in terms of the mesh currents. Be careful in writing the expressions for resistors that are in shared branches.

$$v_{R1} = R_1 \cdot i_a \quad v_{R2} = R_2 \cdot (i_a + i_b) \quad v_{R3} = R_3 \cdot i_a$$

$$v_{R4} = R_4 \cdot i_b \quad v_{R5} = R_5 \cdot i_b$$

6. Substitute the voltage expressions into the KVL equations to create a set of mesh current equations.

$$V_{S1} - i_a R_1 - (i_a + i_b) R_2 - i_a R_3 = 0$$

$$V_{S2} - i_b R_4 - (i_a + i_b) R_2 - i_b R_5 = 0$$

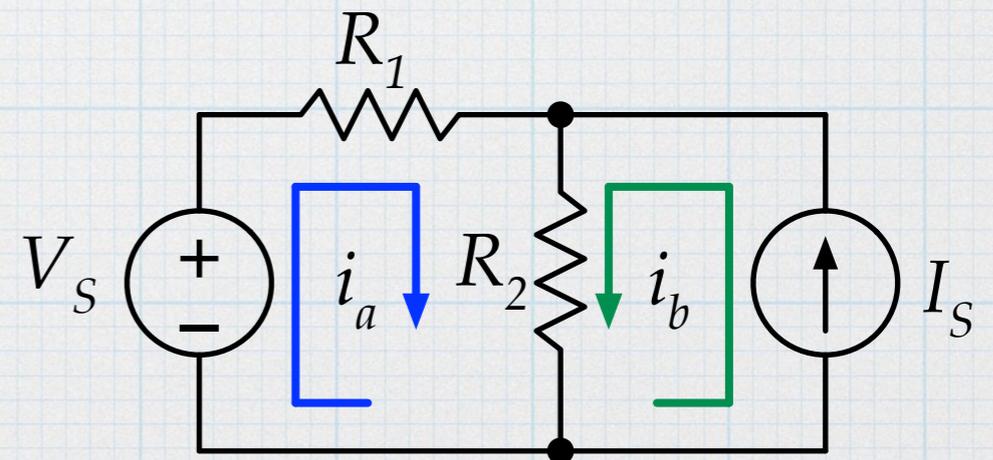
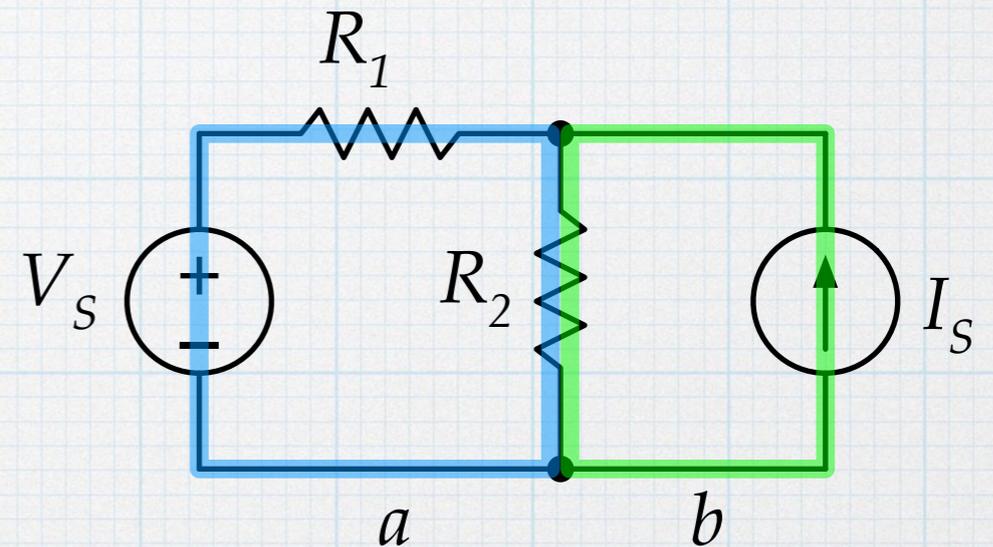
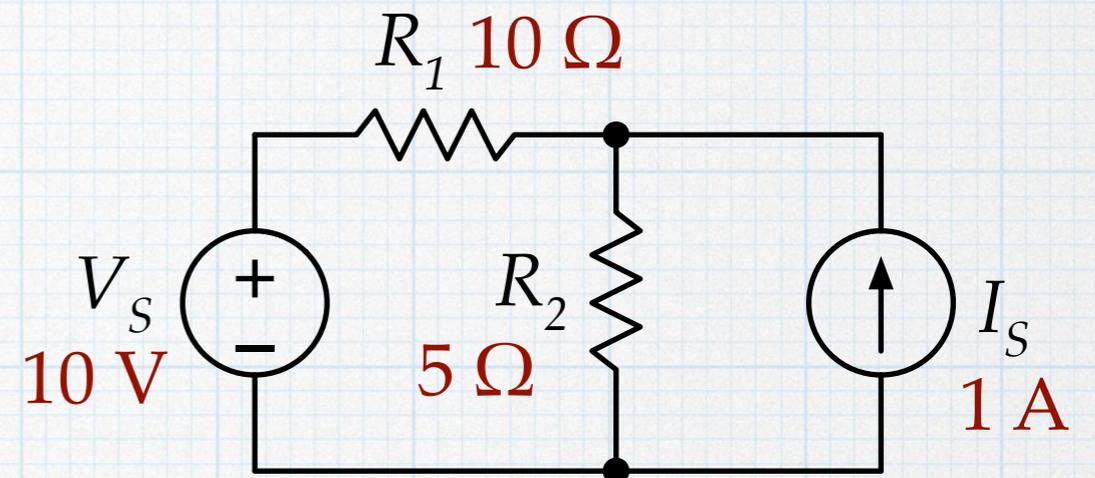
7. The resulting set of simultaneous equations can be solved using your favorite linear algebra techniques.

Example 1

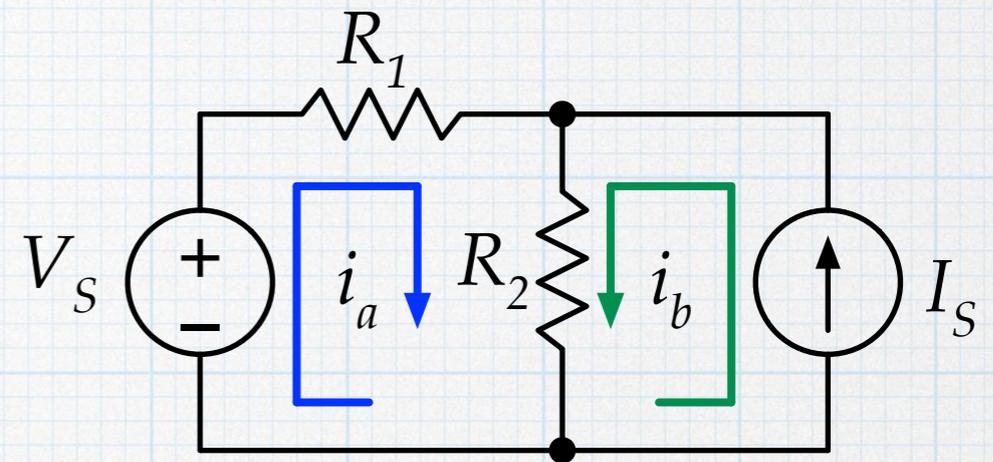
Let's start with an easy one — the familiar two-source, two-resistor circuit. (Of course, we have solved this one previously using the source-transformation and node-voltage methods.)

1. Identify the meshes that define the circuit. Our simple circuit has two meshes, which we label a and b .

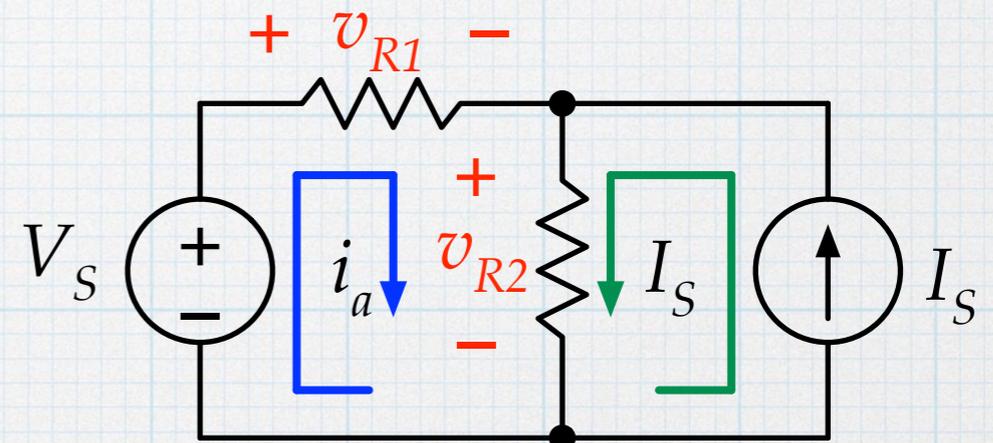
2. Define mesh currents that circulate around each mesh.



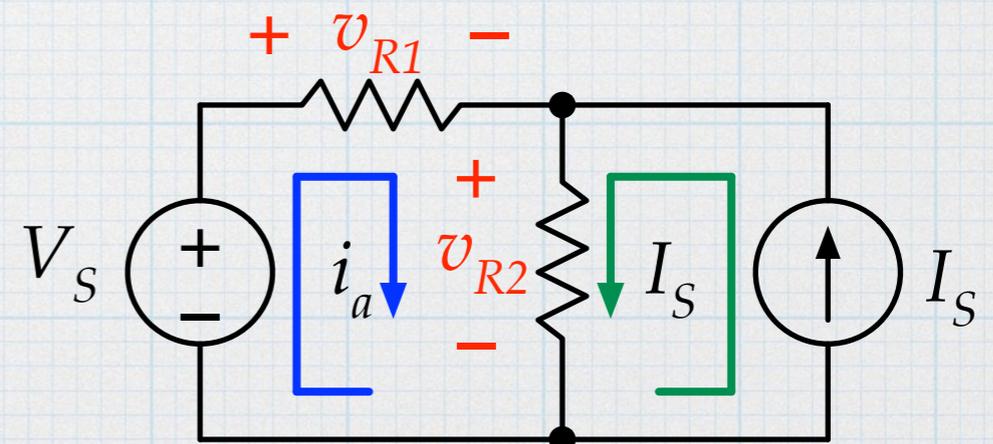
We note something interesting in this case — loop b has a current source in an outside branch. Therefore, i_b must be equal to I_S . We won't need to solve for i_b because we already know it. Also, we see that $i_{V_S} = i_{R_1} = i_a$ and $i_{R_2} = i_a + i_b = i_a + I_S$.



3. Define voltages for all of the components around the meshes where the currents are not known.



4. Write KVL equations around each of the loops where mesh current is not known. There is only one in this case.



$$V_S - v_{R1} - v_{R2} = 0.$$

5. Use Ohm's law to express the resistor voltages in terms of the mesh currents.

$$v_{R1} = R_1 i_a \quad v_{R2} = R_2 (i_a + I_S)$$

6. Substitute the resistor voltage expressions into the KVL equations to form the mesh-current equation. (Just one in this case.)

$$V_S - R_1 i_a - R_2 (i_a + I_S) = 0$$

7. And solve. In this case, there is one equation in one unknown, so it is quite easy.

$$(R_1 + R_2) i_a = V_S - R_2 I_S$$

$$i_a = \frac{V_S - R_2 I_S}{R_1 + R_2} = \frac{10 \text{ V} - (5\Omega)(1 \text{ A})}{10\Omega + 5\Omega} = 0.333 \text{ A}$$

Calculate the resistor voltages, and then we know everything.

$$v_{R1} = R_1 i_a = (10 \Omega)(0.333 \text{ A}) = 3.33 \text{ V}$$

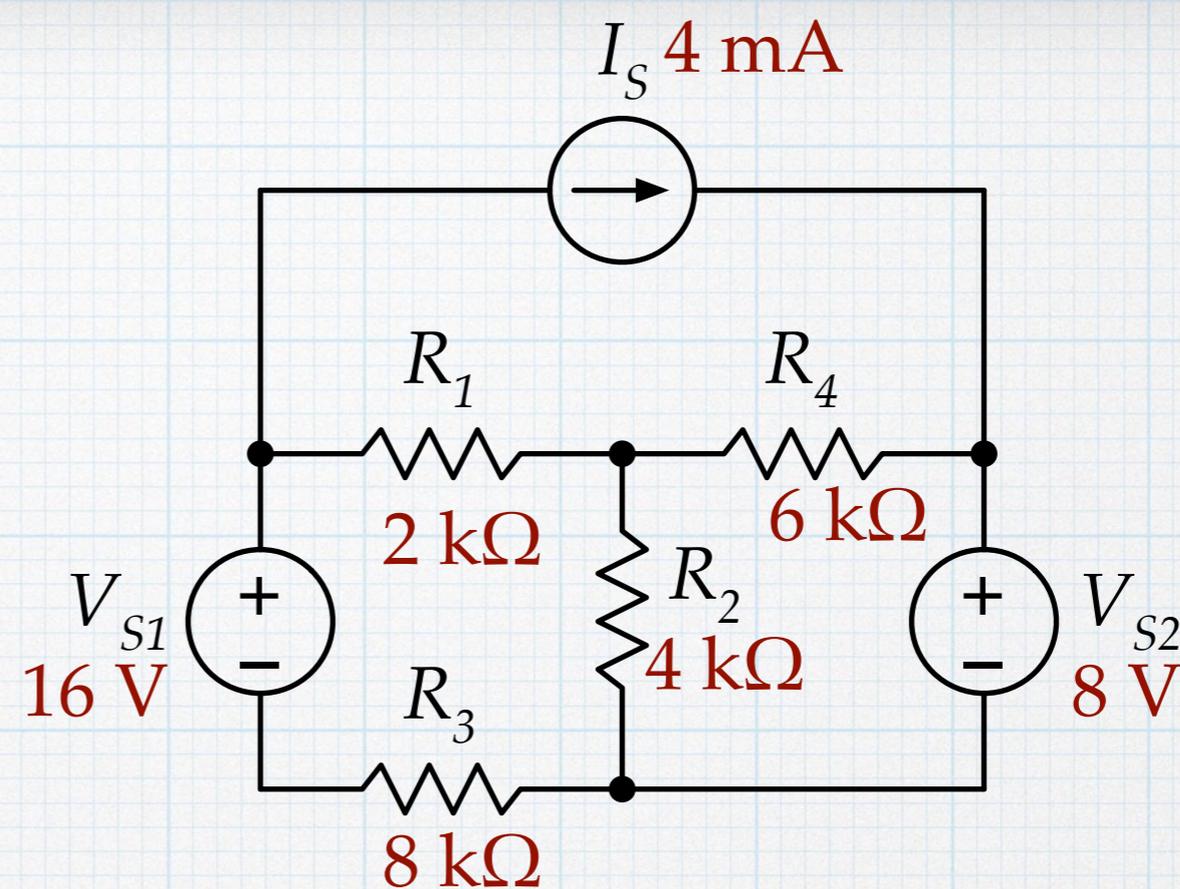
$$v_{R2} = R_2 (i_a + I_S) = (5 \Omega)(0.333 \text{ A} + 1.0 \text{ A}) = 6.67 \text{ V}$$

The mesh current method

1. Identify all of the individual meshes in the circuit.
2. Assign a mesh current to each mesh. Identify meshes in which the current is known because there is a current source in an outside branch of the mesh.
3. Assign voltages to all of the elements in the meshes with unknown currents.
4. Use KVL around each mesh to write loop equation.
5. Use Ohm's law to write the resistor voltages in terms of the mesh currents.
6. Insert the voltage expressions into the KVL equations to arrive at a set of mesh current equations. If there are n unknown currents, there should be n equations relating them.
7. Solve the system of equations to find the mesh currents.

Example 2

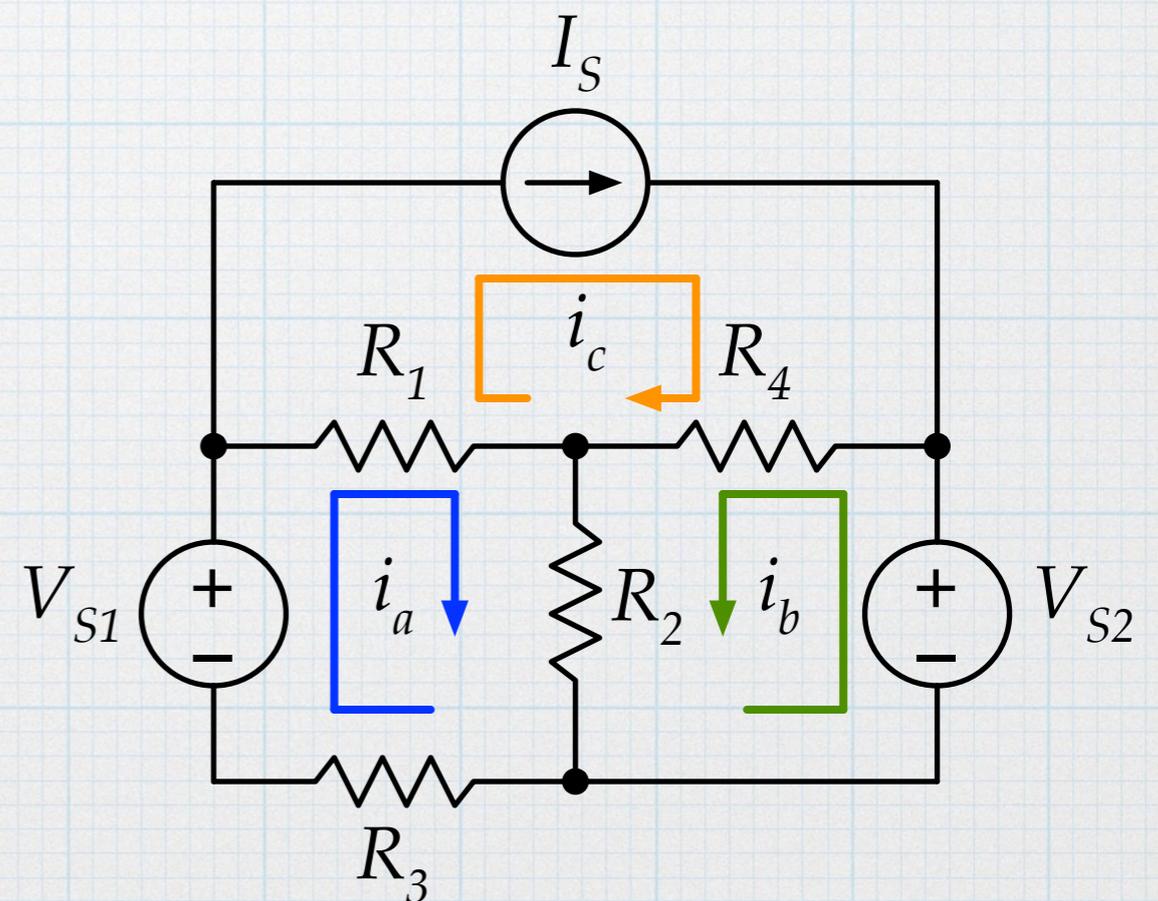
Determine the power being delivered (or absorbed) by V_{S1} .



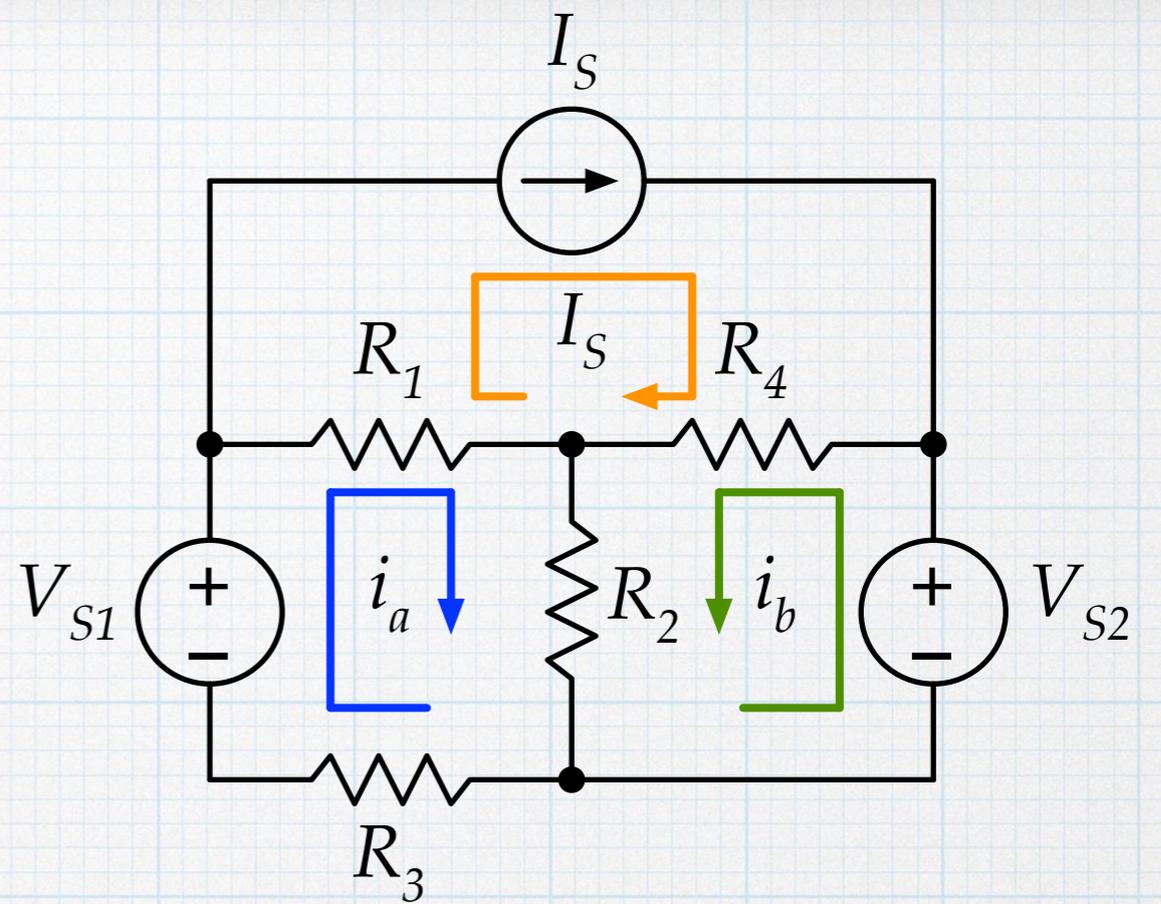
We will need to find $i_{V_{S1}} = I_S + i_{R1}$. The resistor R_3 on the bottom would make the use of node voltages a bit unwieldy, so try mesh currents.

1 & 2. The circuit has three meshes, so define three mesh currents.

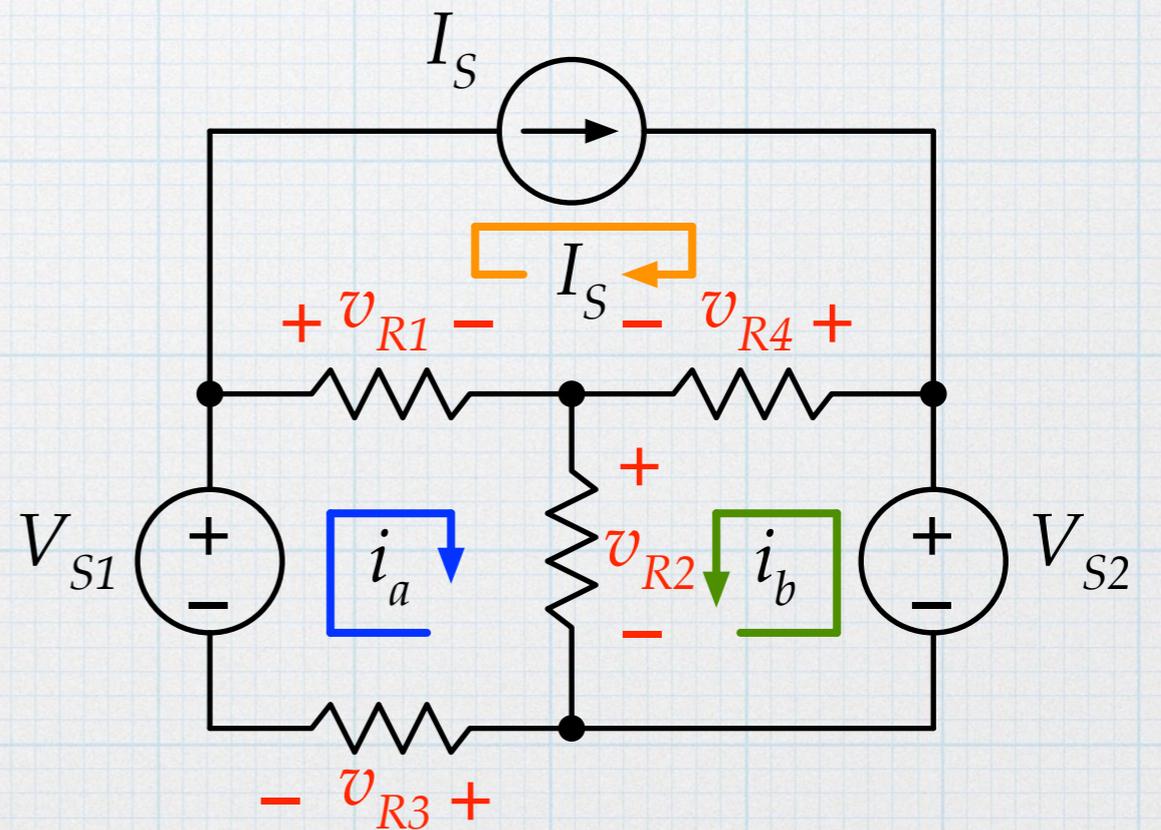
Now note that $i_{V_{S1}} = i_a$.



With the current source in the outside branch of mesh c, we know directly that $i_c = I_S$. That leaves two unknown mesh currents.



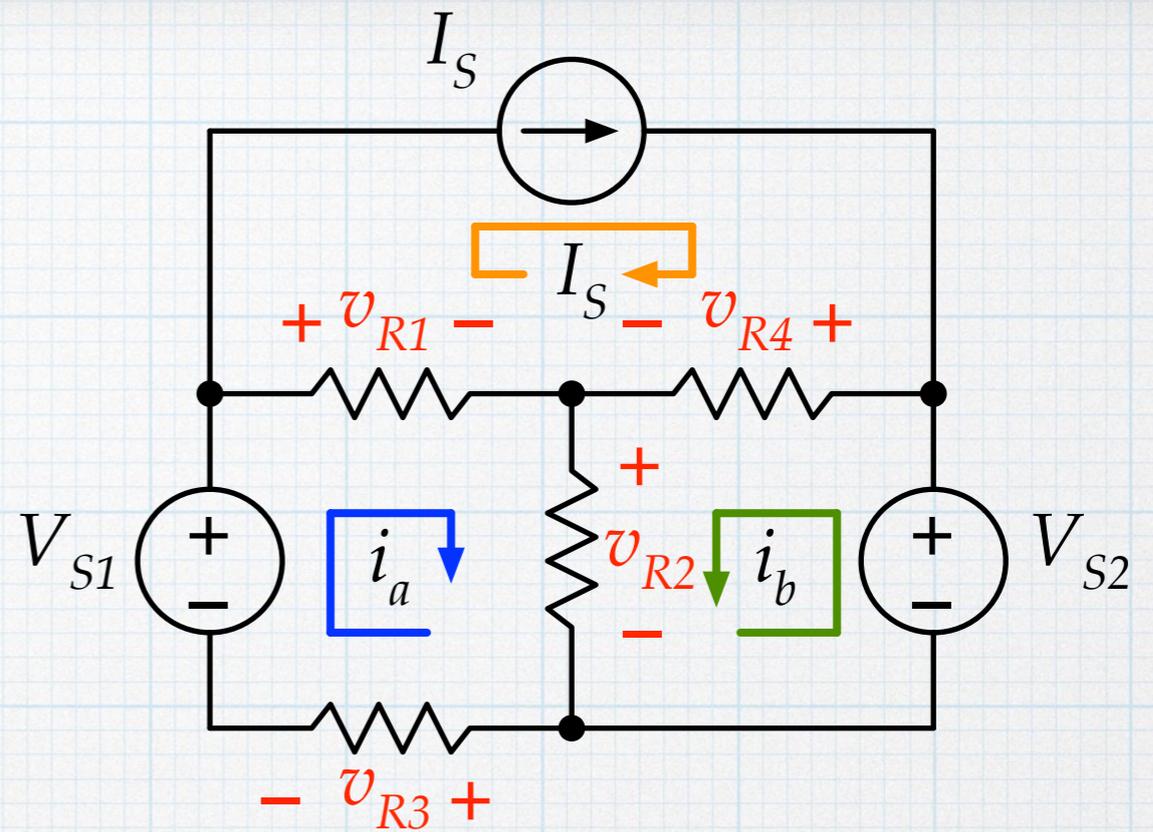
3. Define voltages for each component around each of the unknown meshes.



4. Use KVL to relate the voltages around each mesh.

a: $V_{S1} - v_{R1} - v_{R2} - v_{R3} = 0.$

b: $V_{S2} - v_{R4} - v_{R2} = 0.$



5. Use Ohm's law to write the resistor voltages in terms of the mesh currents. It is important to pay attention to current directions and voltage polarities.

$$v_{R1} = R_1 (i_a - I_S) \quad v_{R2} = R_2 (i_a + i_b) \quad v_{R3} = R_3 i_a \quad v_{R4} = R_4 (i_b + I_S)$$

6. Insert the voltage expressions into the KVL equations to form mesh equations.

a: $V_{S1} - R_1 (i_a - I_S) - R_2 (i_a + i_b) - R_3 i_a = 0.$

b: $V_{S2} - R_4 (i_b + I_S) - R_2 (i_a + i_b) = 0.$

7. Do the math.

$$(R_1 + R_2 + R_3) i_a + R_2 i_b = V_{S1} + R_1 I_S$$

$$R_2 i_a + (R_4 + R_2) i_b = V_{S2} - R_4 I_S$$

$$(2 \text{ k}\Omega + 4 \text{ k}\Omega + 8 \text{ k}\Omega) i_a + (4 \text{ k}\Omega) i_b = 16 \text{ V} + (2 \text{ k}\Omega)(4 \text{ mA})$$

$$(4 \text{ k}\Omega) i_a + (6 \text{ k}\Omega + 4 \text{ k}\Omega) i_b = 8 \text{ V} - (6 \text{ k}\Omega)(4 \text{ mA})$$

$$(14 \text{ k}\Omega) i_a + (4 \text{ k}\Omega) i_b = 24 \text{ V}.$$

$$(4 \text{ k}\Omega) i_a + (10 \text{ k}\Omega) i_b = -16 \text{ V}.$$

$$i_a = 2.45 \text{ mA}.$$

$$i_b = -2.58 \text{ mA}.$$

$$P_{VS1} = V_{S1} i_{VS1}$$

$$= V_{S1} i_a$$

$$= (16 \text{ V})(2.45 \text{ mA}) = 39.2 \text{ mW}$$

Node-voltage or mesh-current?

Deciding which approach to take in a particular circuit usually boils down to determining which method leads to easier math – fewest number of simultaneous equations.

node number (N)

1. Count number of nodes in the circuit.
2. Subtract 1 for ground.
3. Subtract 1 for each voltage source which has a connection (+ or –) to ground.
4. Add 1 for each voltage source which has no connection to ground.

mesh number (M)

1. Count number of meshes in the circuit.
2. Subtract 1 for each current source which is located in an outside branch of a mesh.
3. Add 1 for each current source which is located in an interior branch (shared between two meshes). (More on this later.)

If $N < M$, the node-voltage method should have less math.

If $M < N$, the mesh-current method should have less math.