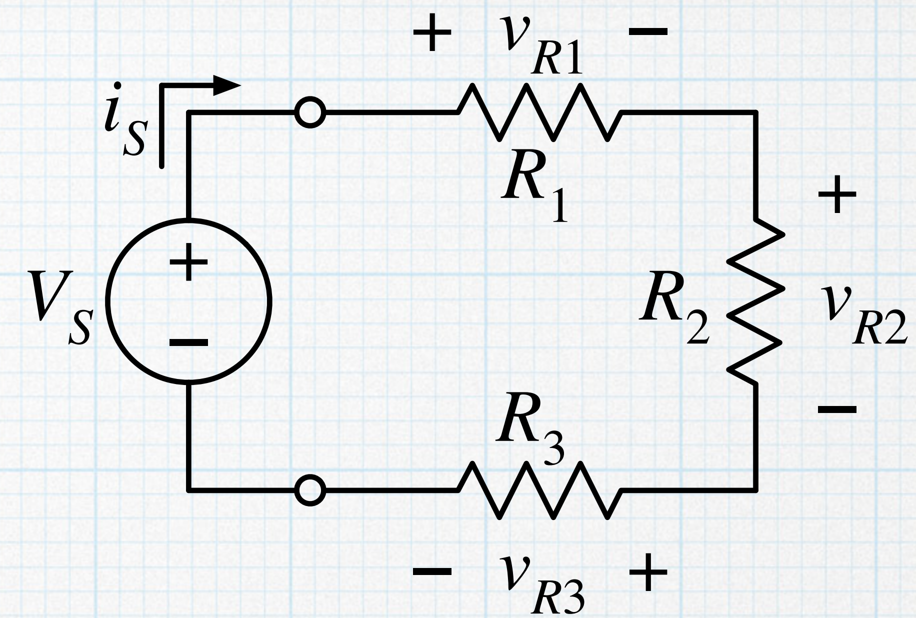
Voltage/current dividers

Voltage and current dividers are easy to understand and use. They are so easy that it may seem not worth the bother of learning them as a separate techniques. But the divider methods, when combined with the equivalent resistances, may be the most used technique in electronics. Knowing how to use dividers will allow us to quickly recognize what is happening in a circuit and determine important voltages and currents.

An engineer could certainly analyze and design circuits without having voltage and dividers in their “tool bag”, but they would be wasting lots of time writing unnecessary KVL and KCL equations.

Voltage divider

Consider a portion of circuit that has several resistors in series, like the circuit at right. Suppose we want to find the voltage across R_2.



We could start by finding the current, which would be equal to the source voltage divided by the equivalent resistance of the string.

$$i_S = \frac{V_S}{R_{eq}}$$

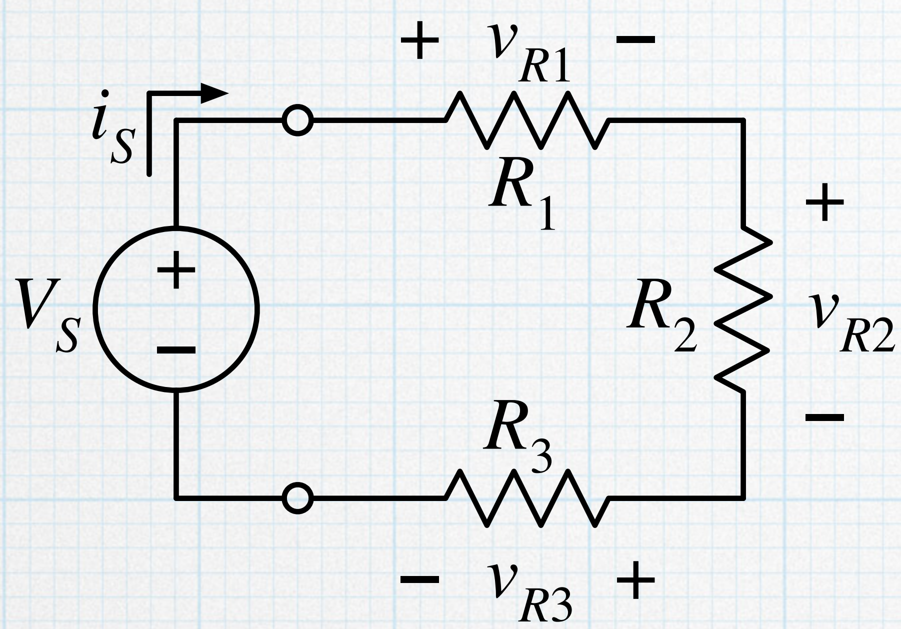
For the series string, $R_{eq} = R_1 + R_2 + R_3$.

Then the voltage across R_2 is just

$$v_{R2} = R_2 \cdot i_S = \frac{R_2}{R_{eq}} \cdot V_S = \frac{R_2}{R_1 + R_2 + R_3} \cdot V_S$$

The total voltage is divided among the resistor in the string. The fraction of the voltage across R_2 is given by a simple resistor ratio.

The other resistor voltages are calculated just as easily.



$$v_{R2} = \frac{R_2}{R_1 + R_2 + R_3} \cdot V_S$$

$$v_{R3} = \frac{R_3}{R_1 + R_2 + R_3} \cdot V_S$$

The three divided voltages sum up to V_S, as KVL insists. If we insert some numbers: V_S = 15 V, R_1 = 4.7 kΩ, R_2 = 15 kΩ, and R_3 = 10 kΩ.

$$v_{R1} = \frac{4.7\,\text{k}\Omega}{4.7\,\text{k}\Omega + 15\,\text{k}\Omega + 10\,\text{k}\Omega}\left(15\,\text{V}\right) = 2.37\,\text{V}$$

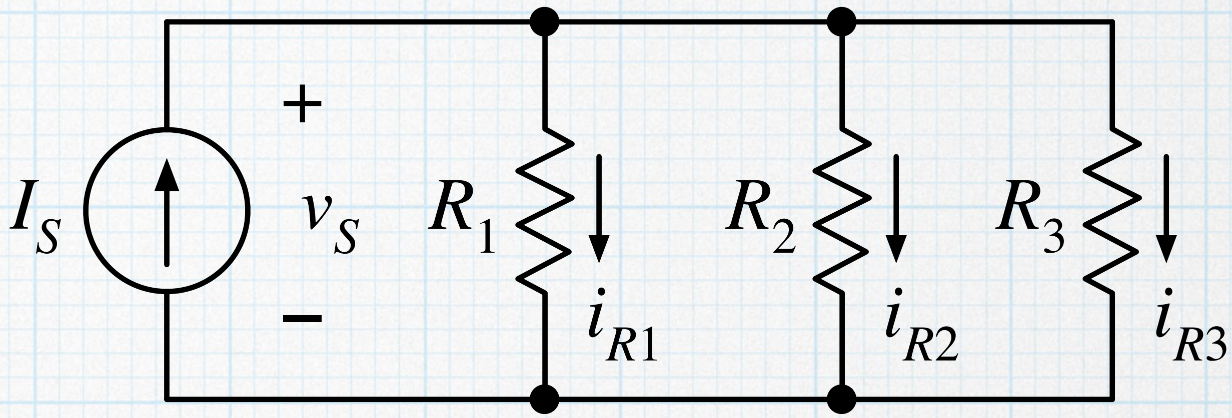
$$v_{R2} = \frac{15\,\text{k}\Omega}{4.7\,\text{k}\Omega + 15\,\text{k}\Omega + 10\,\text{k}\Omega}\left(15\,\text{V}\right) = 7.58\,\text{V}$$

It's that easy.

$$v_{R3} = \frac{10\,\text{k}\Omega}{4.7\,\text{k}\Omega + 15\,\text{k}\Omega + 10\,\text{k}\Omega}\left(15\,\text{V}\right) = 5.05\,\text{V}$$

Current divider

Same idea, but with parallel resistors dividing a current. Suppose we want to know the current through R_2.



We could start by finding the voltage, which would be equal to the source current multiplied by the equivalent resistance of the parallel resistors.

$$v_S = I_S \cdot R_{eq}$$

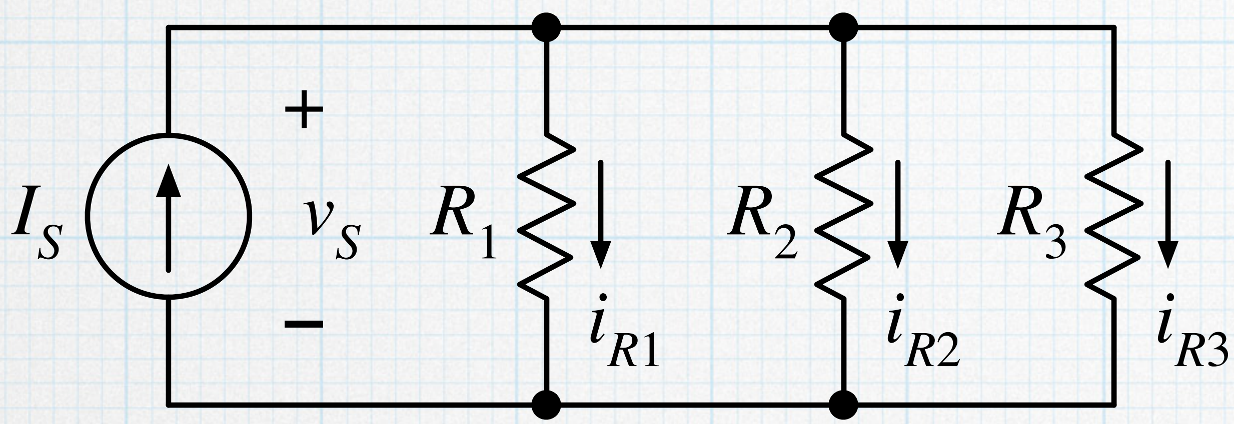
For the parallel combination, $R_{eq} = \dfrac{1}{\dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3}}$

Then the current through R_2 is

$$i_{R2} = \frac{v_S}{R_2} = \frac{R_{eq}}{R_2} \cdot I_S = \frac{\dfrac{1}{R_2}}{\dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3}} \cdot I_S$$

As in the case of the voltage divider, the fraction of the current through one resistor is determined by a simple ratio based on resistor values. But in the current case, resistor inverses are used.

The other resistor currents are calculated just as easily.



$$i_{R1} = \frac{\frac{1}{R_1}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} \cdot I_S$$

$$i_{R3} = \frac{\frac{1}{R_3}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} \cdot I_S$$

The three divided currents sum up to I_S, as KCL insists. If we insert some numbers: $I_S = 15$ mA, $R_1 = 2.2$ kΩ, $R_2 = 3.3$ kΩ, and $R_3 = 6.8$ kΩ.

$$i_{R1} = \frac{\frac{1}{2.2\,\text{k}\Omega}}{\frac{1}{2.2\,\text{k}\Omega} + \frac{1}{3.3\,\text{k}\Omega} + \frac{1}{6.8\,\text{k}\Omega}} \left(15\,\text{mA}\right) = 7.54\,\text{mA}$$

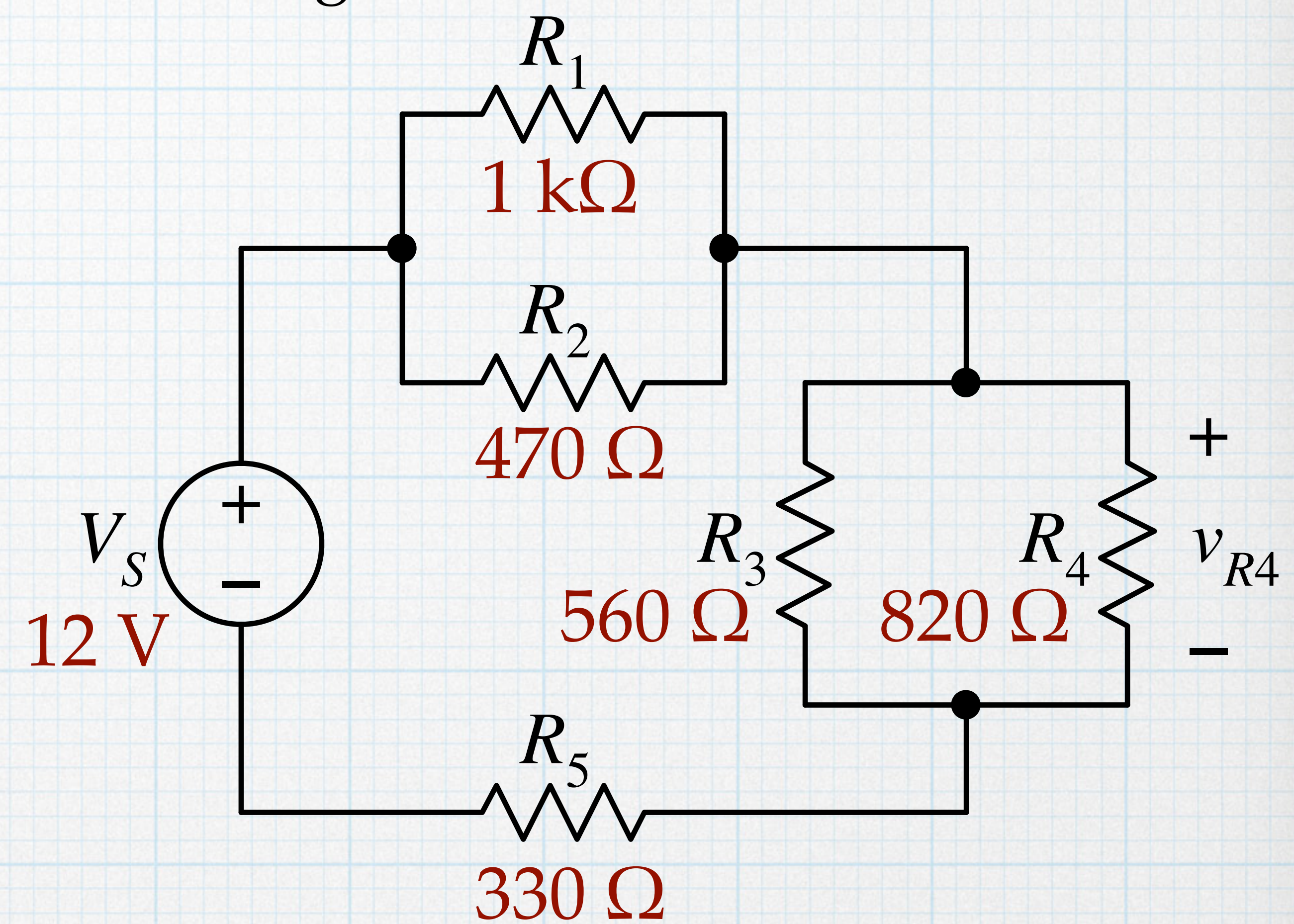
$$i_{R2} = \frac{\frac{1}{3.3\,\text{k}\Omega}}{\frac{1}{2.2\,\text{k}\Omega} + \frac{1}{3.3\,\text{k}\Omega} + \frac{1}{6.8\,\text{k}\Omega}} \left(15\,\text{mA}\right) = 5.02\,\text{mA}$$

$$i_{R3} = \frac{\frac{1}{6.8\,\text{k}\Omega}}{\frac{1}{2.2\,\text{k}\Omega} + \frac{1}{3.3\,\text{k}\Omega} + \frac{1}{6.8\,\text{k}\Omega}} \left(15\,\text{mA}\right) = 2.44\,\text{mA}$$

In many instances, the combination of dividers with equivalent resistances provides for fast calculation of voltages and currents.

Example 1

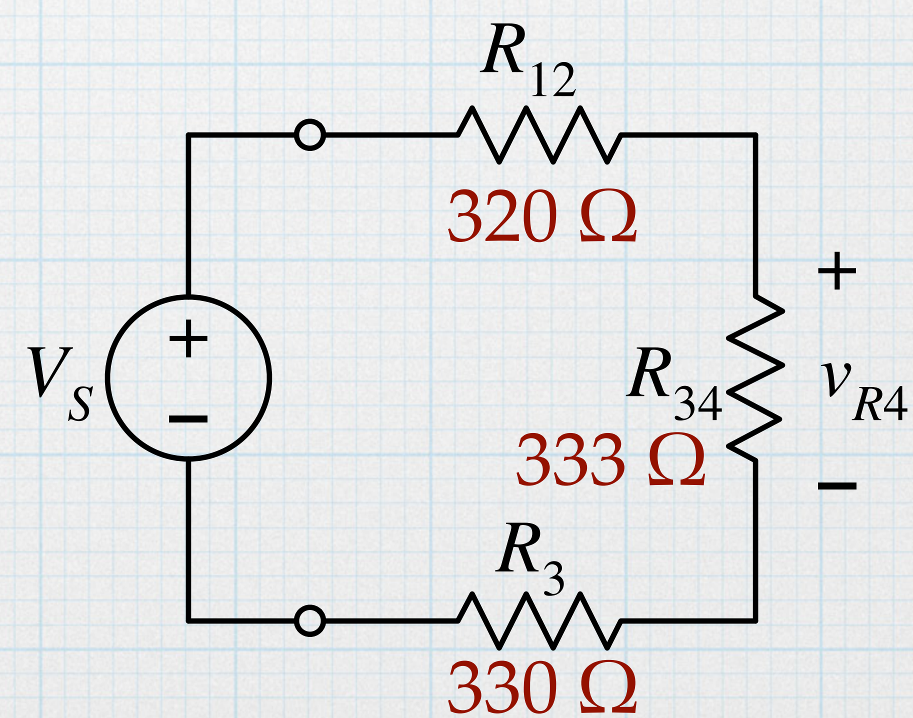
In the circuit at right, find v_{R4}.



Since R_3 and R_4 are in parallel, they have the same voltage and we can use the parallel equivalent. We can also find the parallel equivalent of R_1 and R_2.

$$R_{34} = \frac{R_3 R_4}{R_3 + R_3} = 333\,\Omega$$

$$R_{12} = \frac{R_1 \cdot R_2}{R_1 + R_2} = 320\,\Omega$$

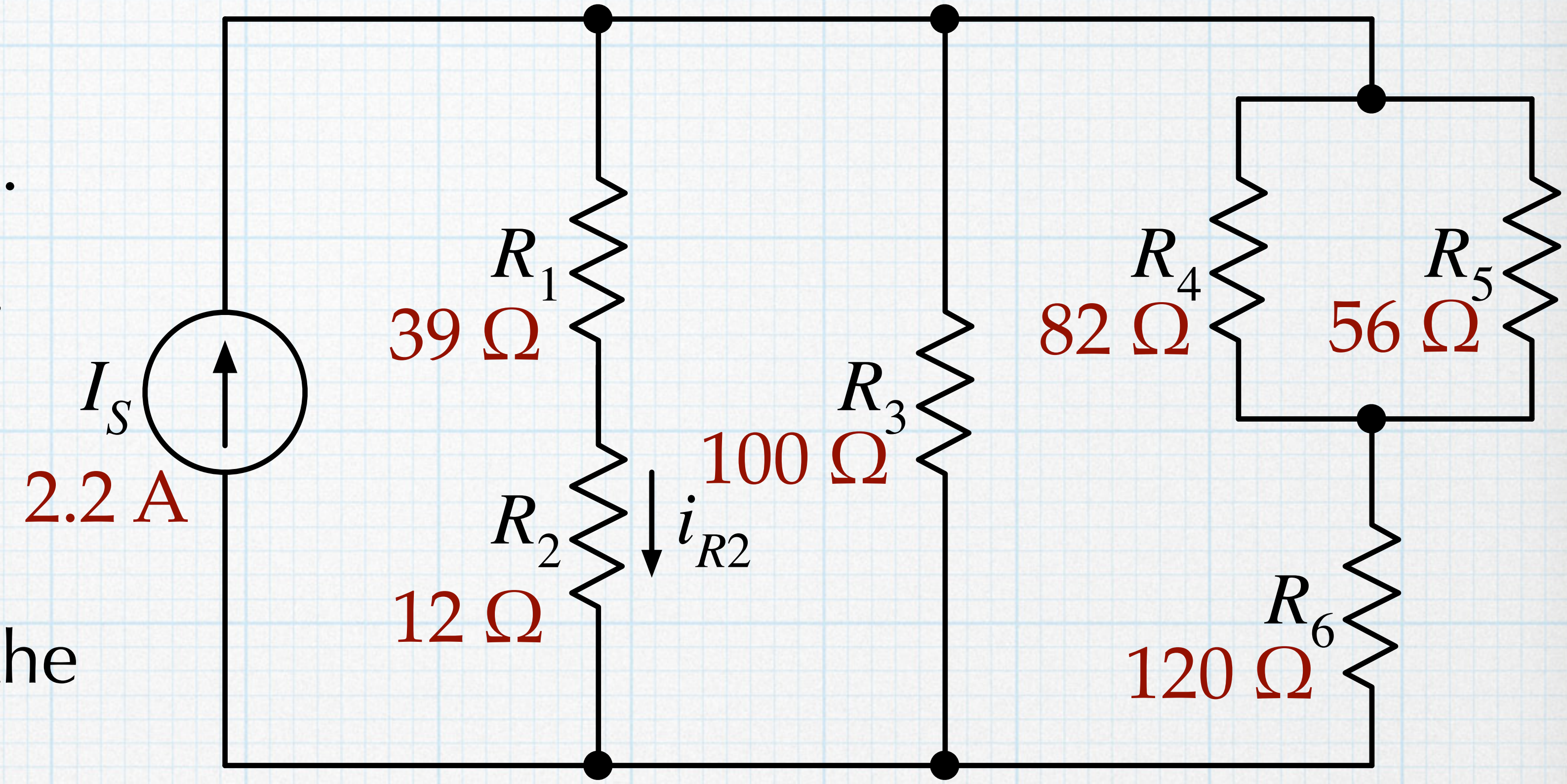


Then use a voltage divider on the simplified circuit.

$$v_{R4} = \frac{R_{34}}{R_{12} + R_{34} + R_5} \cdot V_S = 4.065\text{ V}$$

Example 2

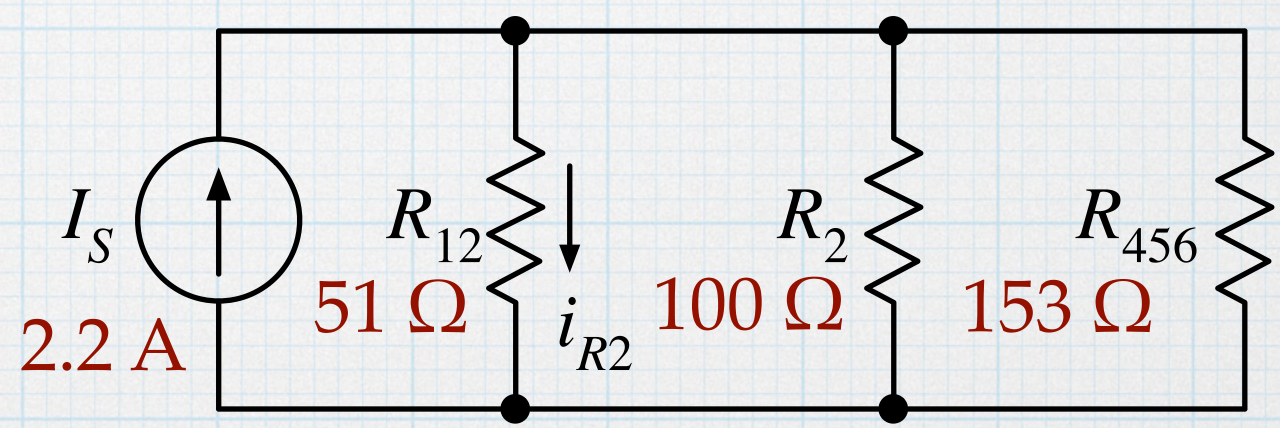
In the circuit at right, find i_{R2}.



Since R_1 and R_2 are in series, they have the same current, and we can use the series equivalent. We can also find the equivalent resistance of the branch with R_4, R_5, and R_6.

$$R_{12} = R_1 + R_2 = 51\,\Omega$$

$$R_{456} = \frac{R_4 \cdot R_5}{R_4 + R_5} + R_6 = 153\,\Omega$$

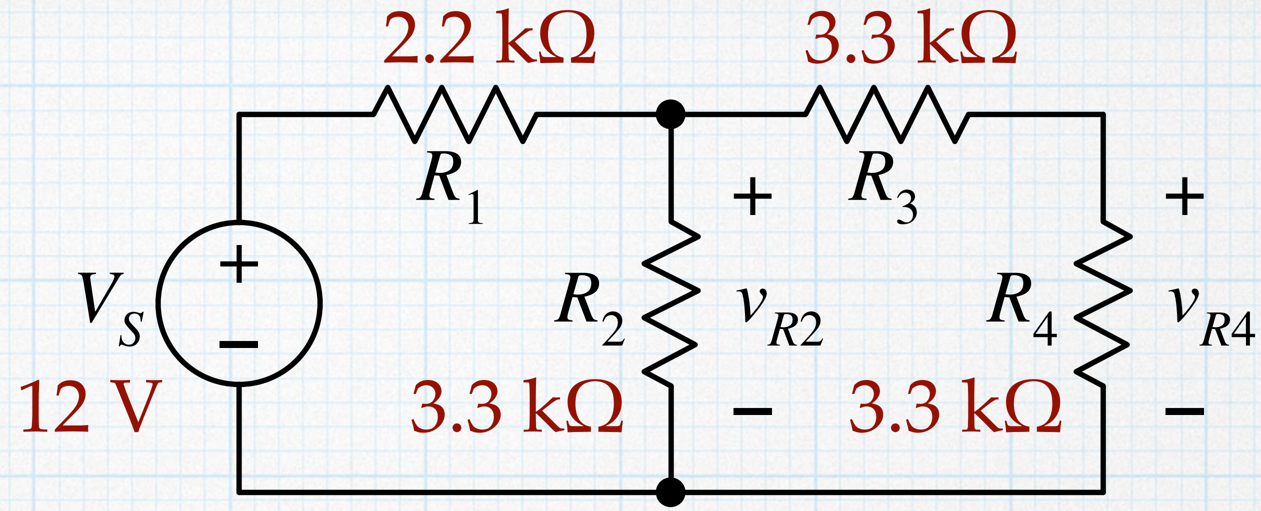


Then use a current divider on the simplified circuit.

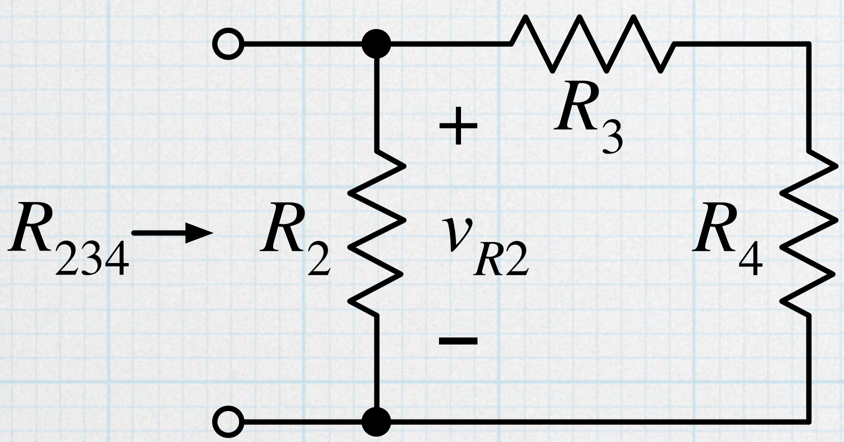
$$i_{R2} = \frac{\frac{1}{R_{12}}}{\frac{1}{R_{12}} + \frac{1}{R_3} + \frac{1}{R_{456}}} \cdot I_S = 1.19\text{ A}$$

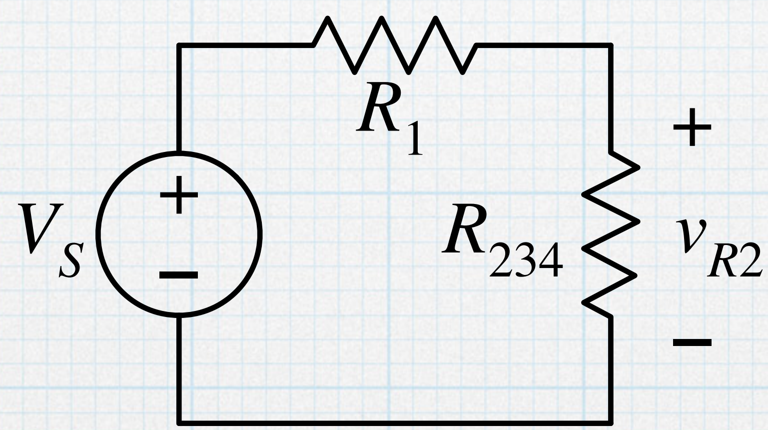
Example 3

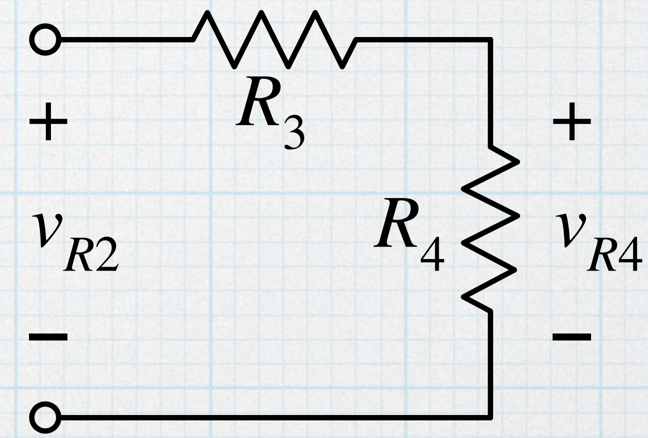
Find v_{R2} and v_{R4} in the circuit.



We solve this using the voltage divider calculation twice in succession. First we find v_{R2} using a voltage divider formed by R_1 and the equivalent resistance of R_2 in parallel with the series combination of R_3 and R_4.







$$R_{234} = R_2 \| (R_3 + R_4)$$

$$= (3.3\,\mathrm{k\Omega}) \| (6.6\,\mathrm{k\Omega})$$

$$= 2.2\,\mathrm{k\Omega}$$

$$v_{R2} = \frac{R_{234}}{R_{234} + R_1} \cdot V_S = 6\ \mathrm{V}$$

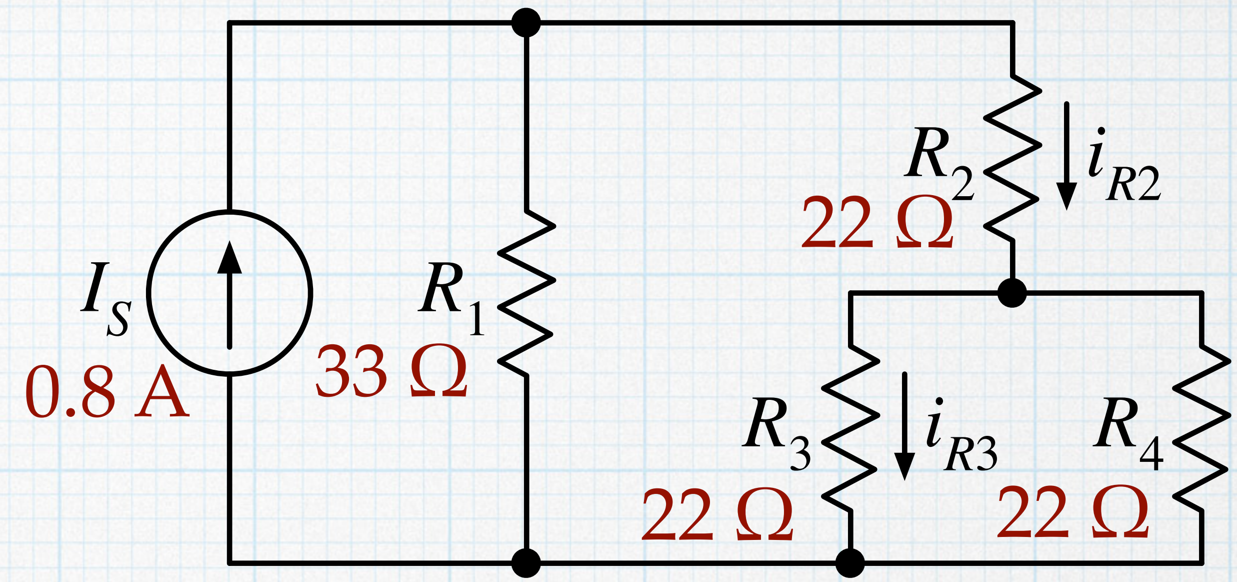
Then the voltage v_{R2} is divided between R_3 and R_4.

$$v_{R4} = \frac{R_4}{R_3 + R_4} \cdot v_{R2} = 3\ \mathrm{V}$$

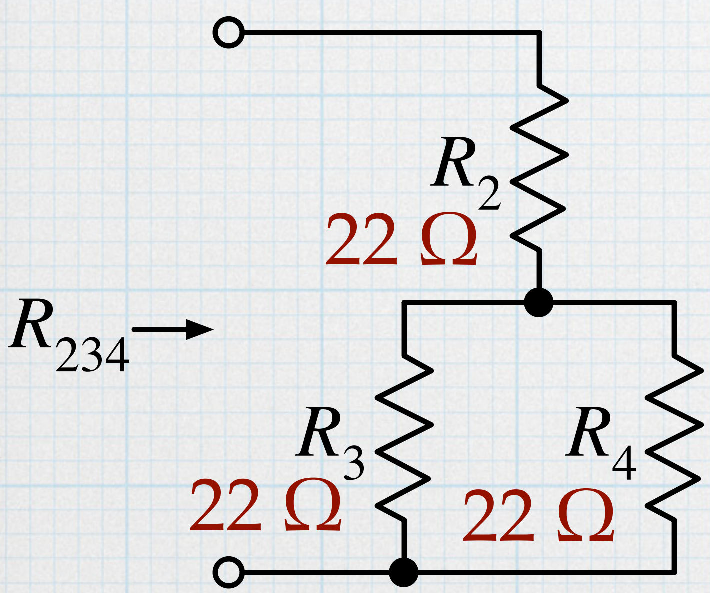
Example 4

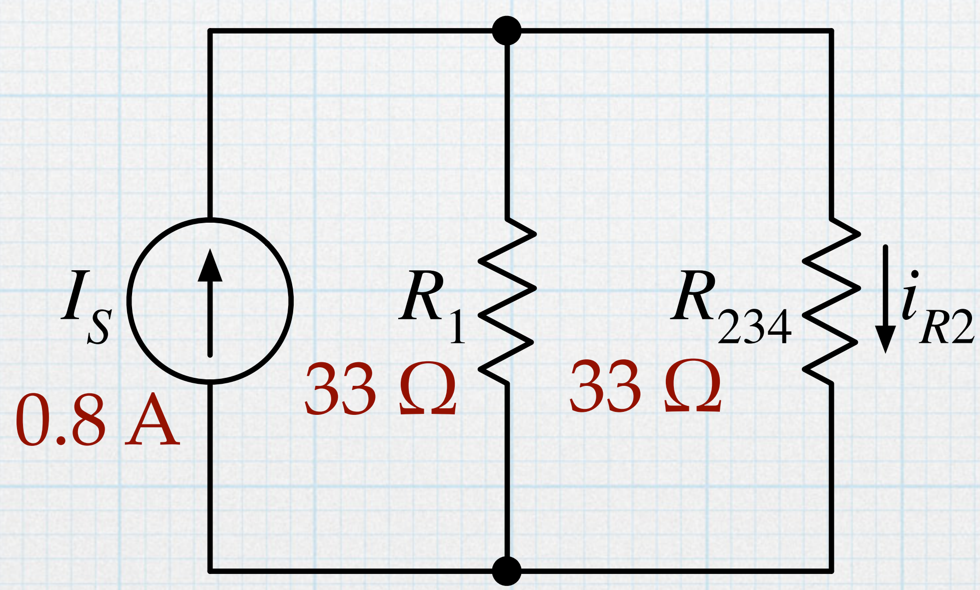
Find i_{R3} in the circuit.

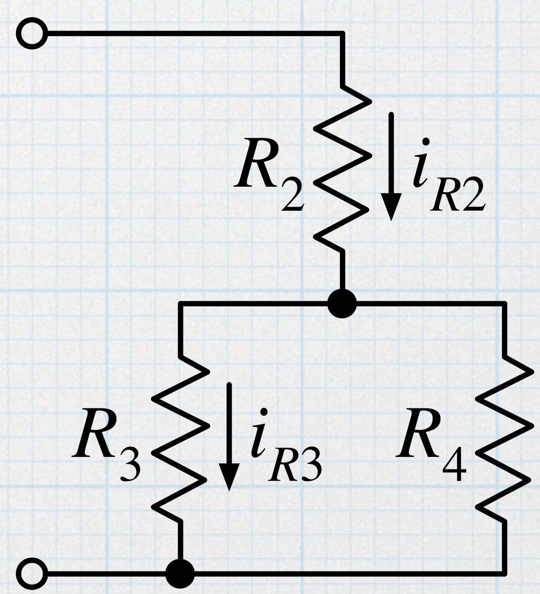
Similar to example 3, we can cascade dividers to find the current in two steps.



I_S splits between R_1 and the branch with R_2, R_3 , and R_4. To find the current through R_2, we use the equivalent resistance of that branch, which forms a current divider with R_1. Then i_{R2} is divided between R_3 and R_4.







$$R_{234} = R_2 + R_3 \| R_4$$

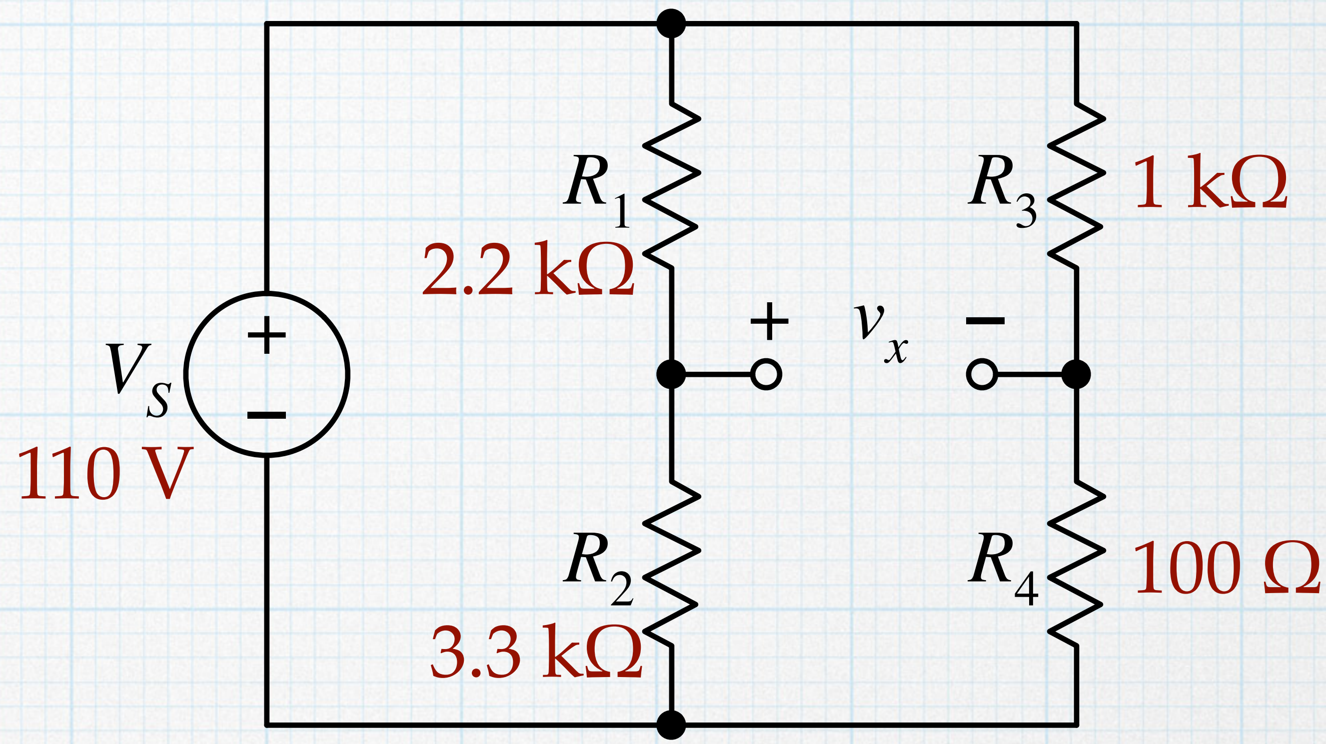
$$= 3.3\,\mathrm{k}\Omega$$

$$i_{R2} = \frac{\frac{1}{R_{234}}}{\frac{1}{R_1} + \frac{1}{R_{234}}} \cdot I_S = 0.4\ \mathrm{A}$$

$$i_{R3} = \frac{\frac{1}{R_3}}{\frac{1}{R_3} + \frac{1}{R_4}} \cdot i_{R2} = 0.2\ \mathrm{A}$$

Example 5

Find the voltage v_x indicated in the circuit at right.



By KVL, $v_x = v_{R2} - v_{R4}$.

We see that R_1 and R_2 form a voltage divider splitting V_S. The same for R_3 and R_4.

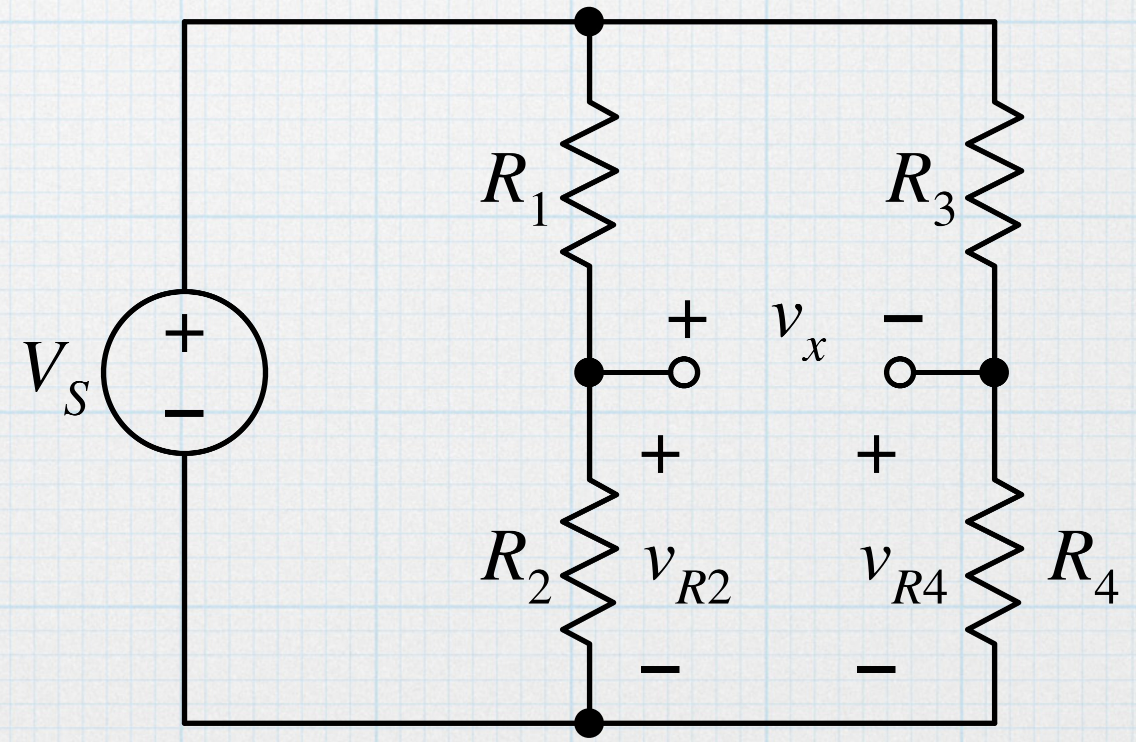
Using voltage dividers.

$$v_{R2} = \frac{R_2}{R_1 + R_2} \cdot V_S = 66 \text{ V}$$

$$v_{R4} = \frac{R_4}{R_3 + R_4} \cdot V_S = 10 \text{ V}$$

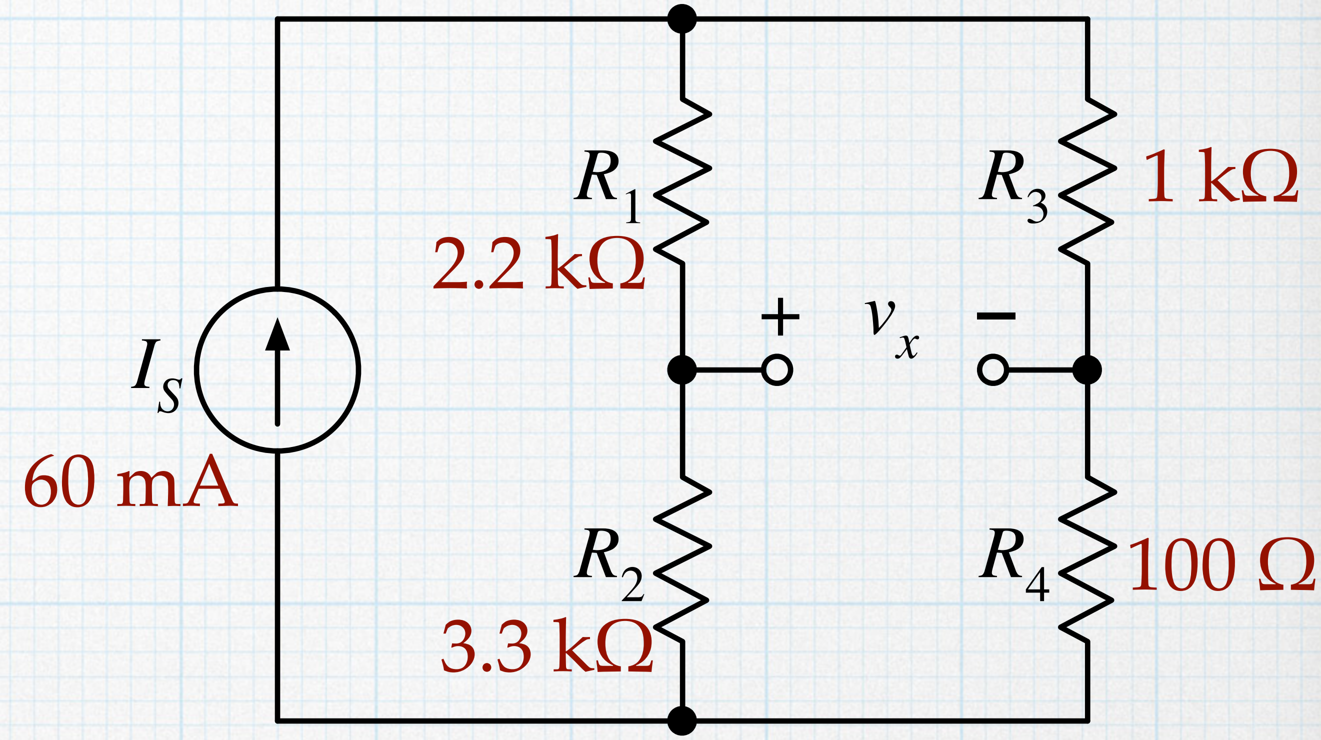
Then

$$v_x = 66 \text{ V} - 10 \text{ V} = 56 \text{ V}.$$



Example 6

Find the voltage v_x indicated in the circuit at right.



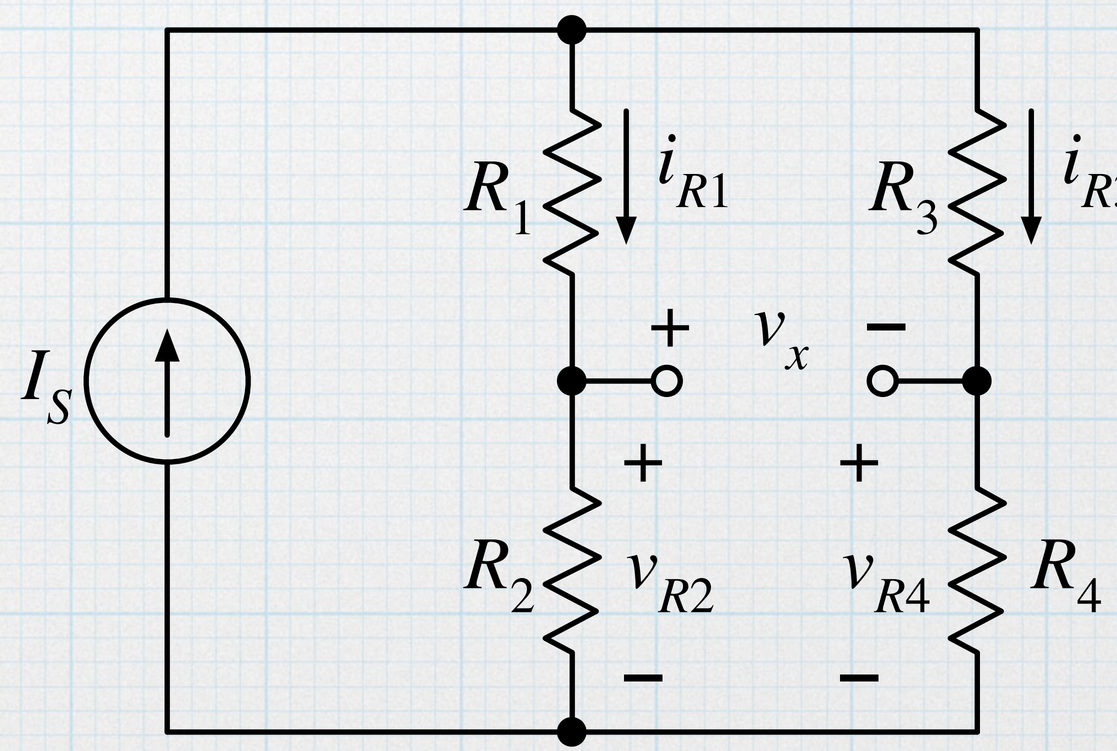
As with the previous example, KVL tells us that $v_x = v_{R2} - v_{R4}$.

We see that the series combination $R_1 + R_2$ forms a current divider with the series combination $R_3 + R_4$, splitting I_S between the two branches.

Using current dividers.

$$i_{R1} = \frac{\frac{1}{R_1 + R_2}}{\frac{1}{R_1 + R_2} + \frac{1}{R_3 + R_4}} \cdot I_S = 10\,\text{mA}$$

$$i_{R3} = \frac{\frac{1}{R_3 + R_4}}{\frac{1}{R_1 + R_2} + \frac{1}{R_3 + R_4}} \cdot I_S = 50\,\text{mA}$$

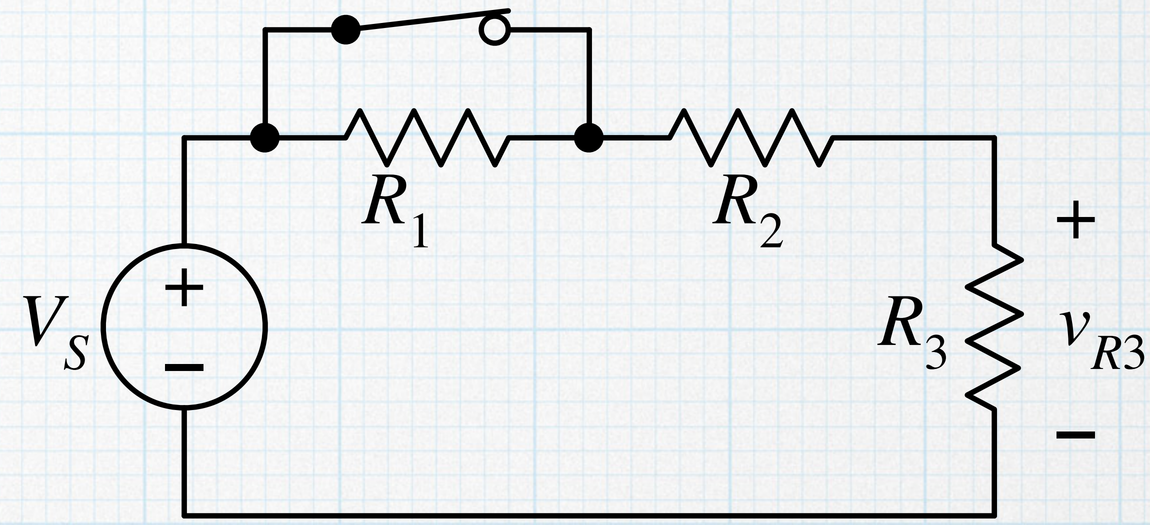


Then

$$v_x = i_{R1} R_2 - i_{R3} R_4 = 28\text{ V}.$$

Example 7

In the circuit at right, the switch can be opened or closed to control the voltage across R_3. When the switch is closed (R_1 shorted out), v_{R3} is twice as big as the case when the switch is open (R_1 not shorted.) How is R_1 related to $R_2 + R_3$?



There are several approaches to answering this question, but using voltage dividers is a convenient method. With the switched closed, R_1 is shorted out and

$$v_{R3} = \frac{R_3}{R_2 + R_3} \cdot V_S$$

With the switch open, R_1 is part of the divider:

$$v'_{R3} = \frac{R_3}{R_1 + R_2 + R_3} \cdot V_S$$

$$v_{R3} = 2v'_{R3}$$

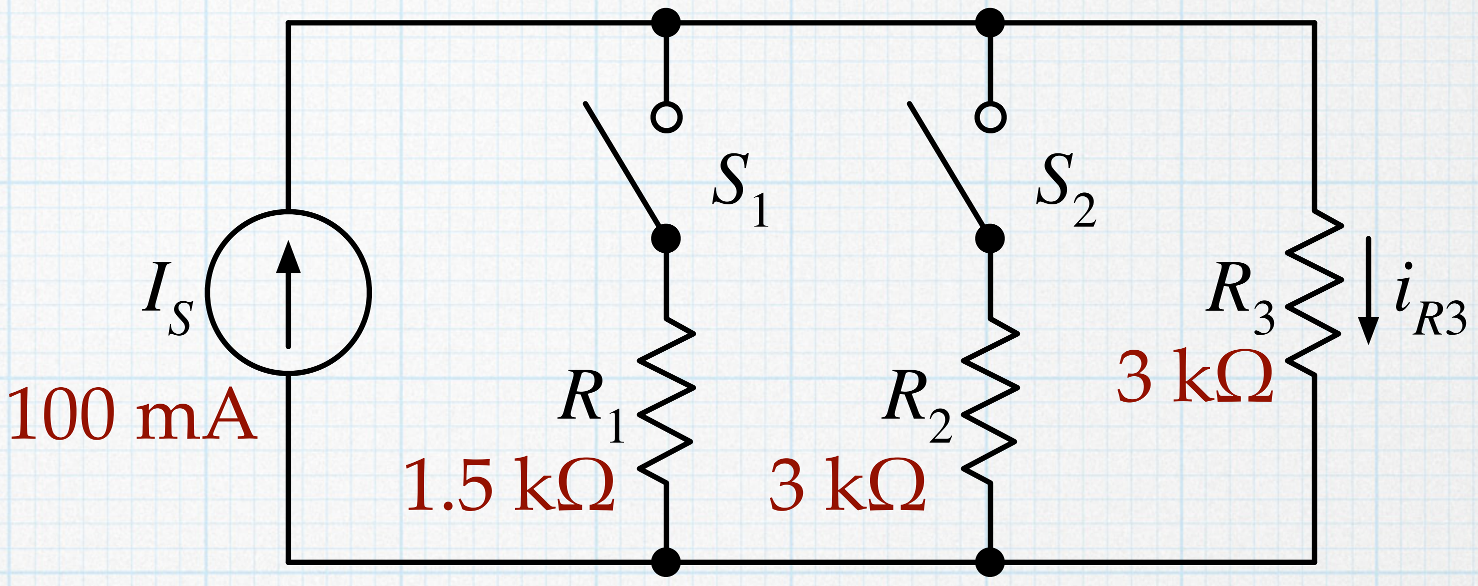
$$\frac{R_3}{R_2 + R_3} \cdot V_S = \frac{2R_3}{R_1 + R_2 + R_3} \cdot V_S$$

$$R_1 + R_2 + R_3 = 2\left(R_2 + R_3\right)$$

$$R_1 = R_2 + R_3$$

Example 8

In the circuit at right, the two switches can be opened or closed to control the current through R_3. Calculate the current through R_3 for all combinations of the switches being open closed.



S_1 open and S_2 open: $i_{R3} = I_S = 100\text{ mA}$

S_1 open and S_2 closed: $i_{R3} = \dfrac{\frac{1}{R_2}}{\frac{1}{R_3} + \frac{1}{R_3}} \cdot I_S = 50\,\text{mA}$

S_1 closed and S_2 open: $i_{R3} = \dfrac{\frac{1}{R_3}}{\frac{1}{R_1} + \frac{1}{R_3}} \cdot I_S = 33.3\,\text{mA}$

S_1 closed and S_2 closed: $i_{R3} = \dfrac{\frac{1}{R_3}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} \cdot I_S = 25\,\text{mA}$