Diodes

Diodes are two-terminal electronic devices, made out of a semiconductor materials. Silicon is the most common semiconductor, but there are others, like germanium, gallium arsenide (GaAs), indium phosphide (InP), gallium nitride (GaN), and many others.

Semiconductors are interesting because their electrical properties can be varied over many order of magnitude: resistivity as high as 10^7 Ω-m (almost an insulator) or as low as 10^{-6} Ω-m (almost a conductor).

Also, semiconductors can be made in two different "varieties": either n-type in which current is carried by electrons – as usual – or *p-type,* in which current behaves like it is carried by positive charges, which we cal "holes".

A diode consists of a layer of n-type joined to a layer of *p*-type material creating a *p-n junction*. When a voltage is applied across the junction, a current flows in response, but the *i-v* relationship is extremely non-linear. The non-linearity will force us to modify our approach to analyzing circuits.

Diode applications

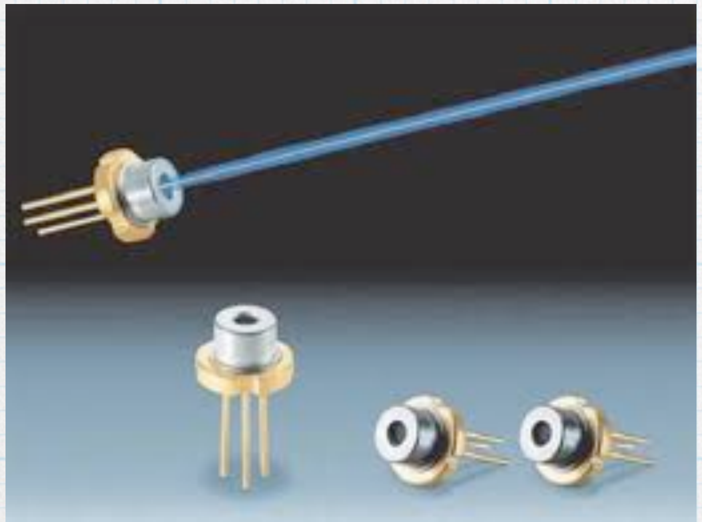
- Rectification – cutting off the top half or bottom half of a voltage signal.
- Voltage regulation – providing a steady voltage reference in a circuit.
- light-emitting diodes – for indicators
- light-emitting diodes – for illumination
- lasers - DVD players, fiber-optic communication, surgery
- photodetectors – sense presence of light, especially low levels or fast pulses
- photovoltaics (solar cells) – "green" electrical power generation
- building block for transistors

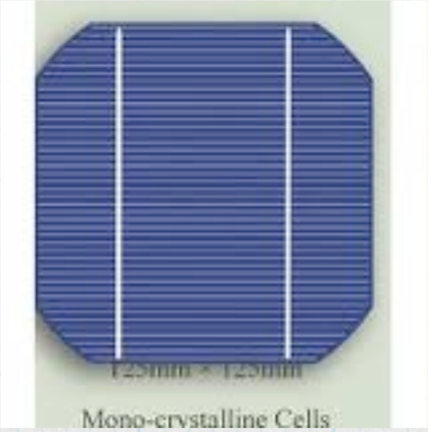
rectifying diode
(switching or
small-signal)

made of silicon

LEDs – various materials (not silicon).
Different material = different colors.

laser

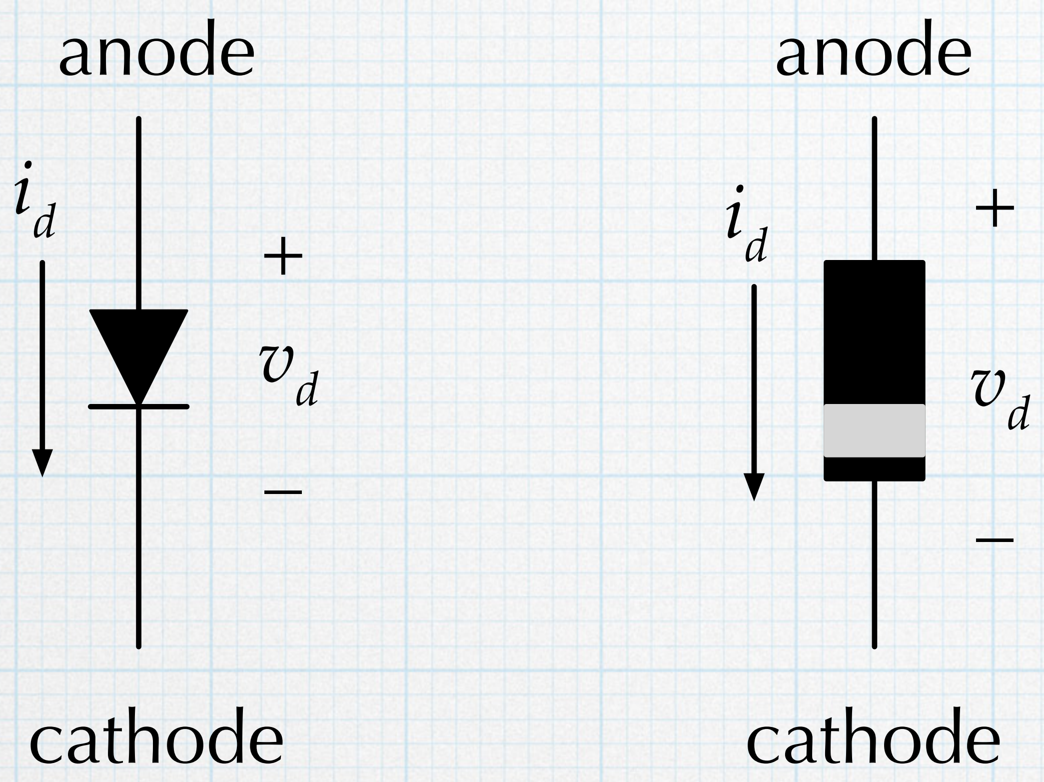


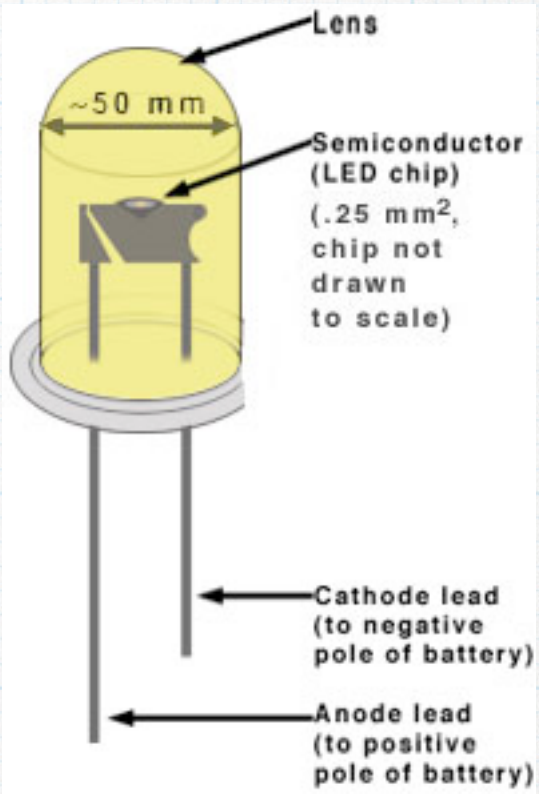


solar cell

LED lighting – usually gallium nitride
(UV light) that excites a phosphor.

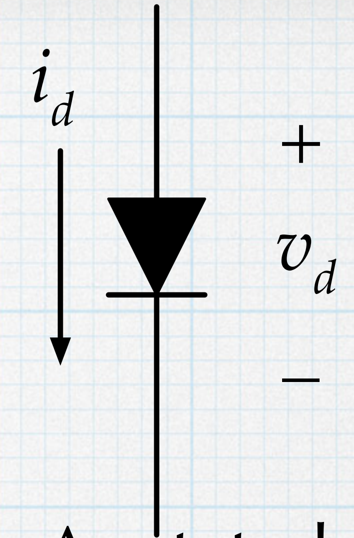
Diode





think "funnel" – current flows in one direction

diode i-v characteristic



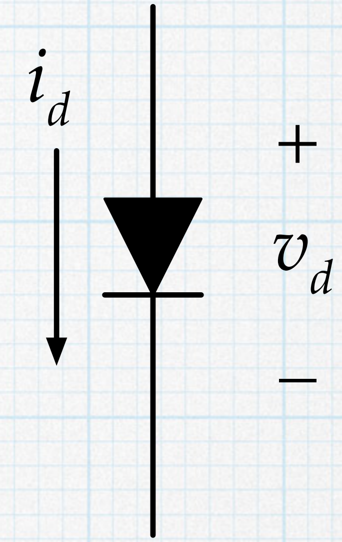
The ideal diode equation: $$i_D = I_S \left[\exp\left(\frac{v_D}{{}^{kT}/_{q}} \right) - 1 \right]$$

where i_D is the diode current and v_D voltage across the diode. As stated earlier, the relationship is extremely non-linear, and it will cause us a some grief when analyzing diodes. But the non-linear behavior offers opportunities for new applications.

- I_S is the current parameter of the diode, often known as the *saturation current* or *scale current*. It is like "R" for a resistor. Each diode will have a unique value for I_S. A typical value is $I_S \approx 10^{-14}$ A.
- kT/q is the *thermal voltage*. k is Boltzmann's constant (recall thermodynamics from physics) with a value of 1.38×10^{-23} J/K. T is the absolute temperature of the diode, expressed in kelvin (K). Then the product kT is the thermal energy and represents the average energy of an electron in the semiconductor. If we divide the electron the electron charge — $q = 1.6 \times 10^{-19}$ C — we get the thermal voltage. At 300 K (= 27°C, approximately room temperature), $kT/q = 25.8$ mV.

diode: forward and reverse conduction

$$i_D = I_S \left[\exp\left(\frac{v_D}{kT/q} \right) - 1 \right]$$



If v_D is more positive than about $3 \cdot kT/q$ (≈ 75 mV at room temperature.)

$$i_D \approx I_S \exp\left(\frac{v_D}{kT/q} \right)$$

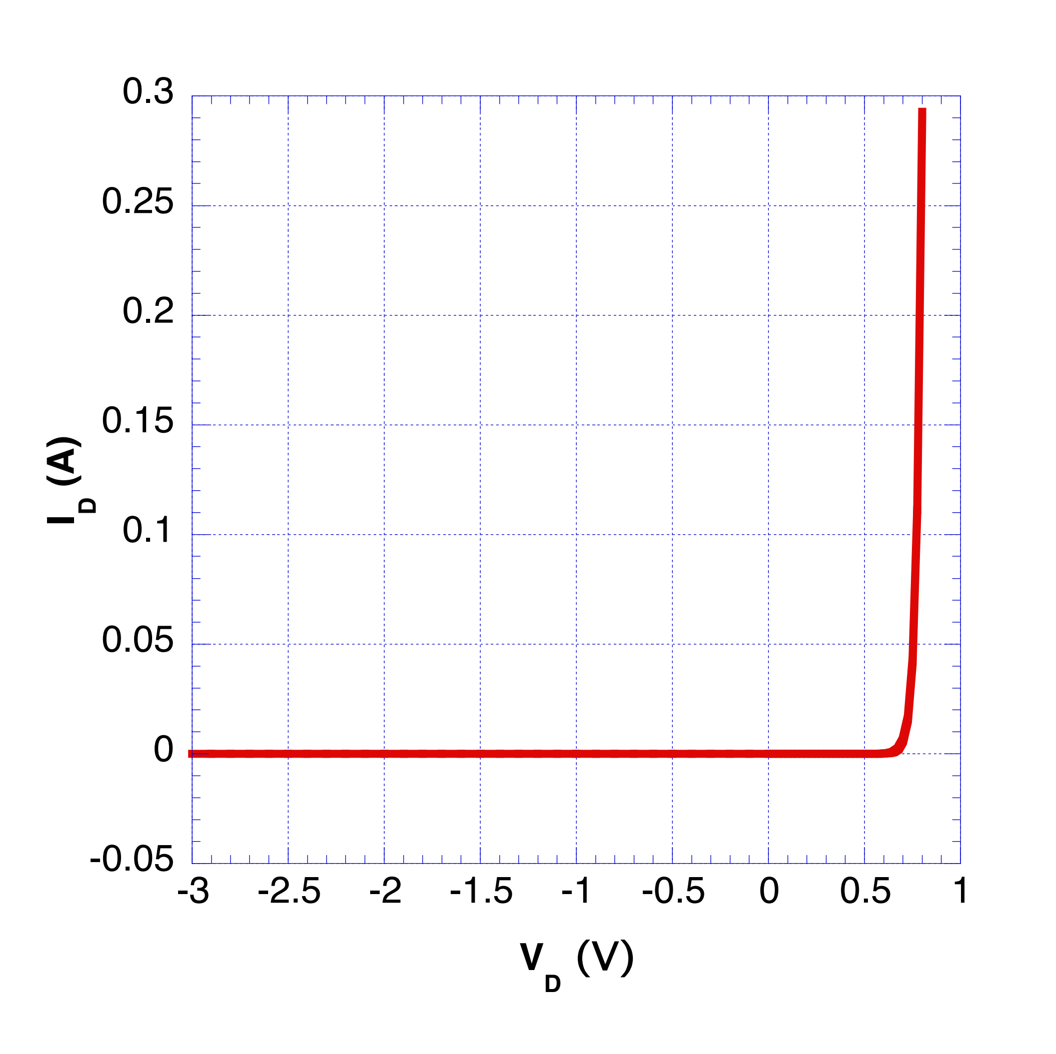
Current increases exponentially with increasing voltage. This is f*orward bias* or *forward conduction*.

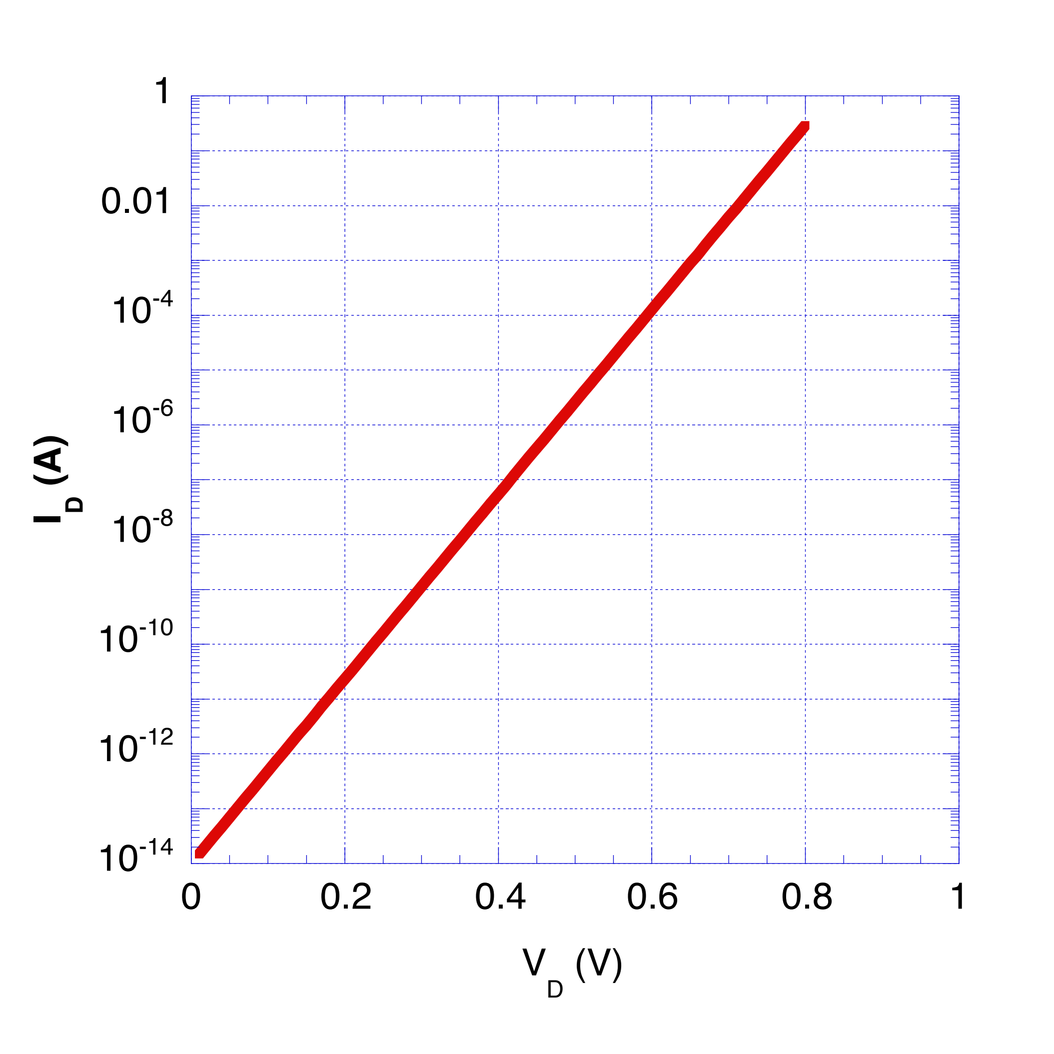
If v_D is more negative than about $-3 \cdot kT/q$

$$i_D \approx -I_S$$

A very small trickle of current flows — almost zero. The current is independent of voltage. This is *reverse bias*.

In an ideal diode, current essentially flows in only one direction. This is asymmetry is the basis for some of the important applications of diodes. (We will see later that current *can* flow in the reverse direction in the right circumstances.





Diode *i-v*
$I_S = 10^{-14}$ A
$T = 300$ K

Same diode
Forward voltage only
semi-log plot

diodes in circuits

The non-linear behavior has some significant effects:

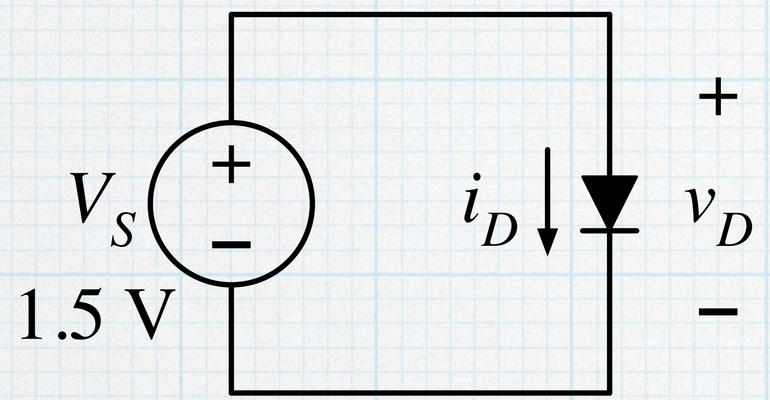
- Basic notions are still valid: KCL and KCL, energy, power.
- Node-voltage and mesh-current techniques are still applicable, but they usually result in a set of non-linear equations, which are difficult to solve.
- Techniques that rely on linearity — superposition and Thevenin equivalents — cannot be applied directly when non-linear elements are present.

When analyzing circuits with non-linear elements, we will make use of:

- Approximations that simplify the non-linear behavior. Essentially we will linearize the elements — hiding the exponential behavior and allowing us to use our familiar techniques. This approximation technique requires some initial guesses and the results must be checked to confirm. Of course, it is only approximate.
- SPICE. It was invented to handle circuits with non-linear elements.

diodes in circuits

Important: When working with diodes, don't EVER apply a forward voltage directly across the diode. The result is usually a dead diode.



$I_S = 10^{-14}$ A
room temp: $kT/q = 25.8$ mV.

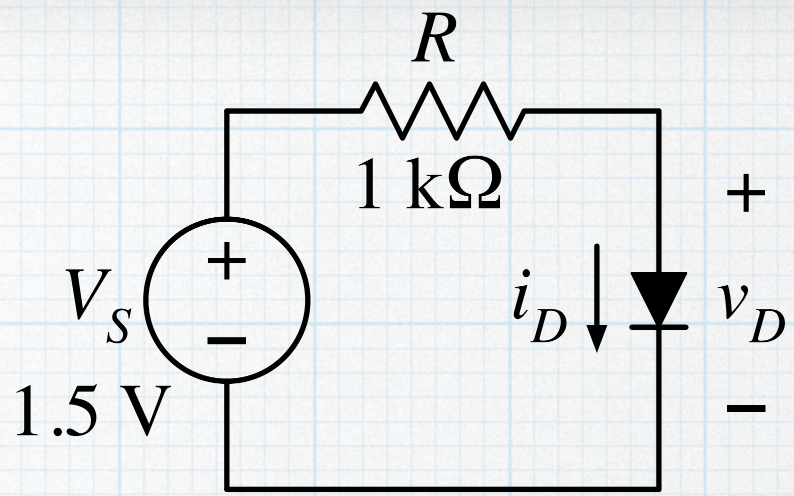
$$v_D = V_S$$

$$i_D \approx I_S \exp\left(\frac{v_D}{kT/q}\right)$$

$$= \left(10^{-14}\,\text{A}\right)\exp\left[\frac{1.5\,\text{V}}{0.0258\,\text{V}}\right] = 1.8 \times 10^{11}\,\text{A} = 180\,\text{GA}$$

Of course, this is absolutely absurd. What really happens is that the diode would rapidly heat up and burn out during the transient as the current increased. There must always be something — probably a resistor — to limit the current.

So add a current-limiting resistor in series.



$$V_S - v_R + v_D = 0$$

$$V_S - i_D R - v_D = 0$$

$$i_D = I_S \left[\exp\left(\frac{v_D}{kT/q} \right) - 1 \right] \rightarrow v_D = \frac{kT}{q} \ln \left[\frac{i_D}{I_S} + 1 \right]$$

$I_S = 10^{-14}$ A.
$kT/q = 25.8$ mV.

$$V_S - i_D R - \frac{kT}{q} \ln \left[\frac{i_D}{I_S} + 1 \right] = 0$$ Yikes!

The result is a *transcendental equation*. It is a perfectly valid relationship for which there is a unique value for the current, but we can't solve it by usual algebraic techniques. It is impossible. To find the the current, we are forced to use numerical techniques, meaning that are we will use a sequence of smart "trial-and-error" steps to determine the value of i_D. Numerical analysis is an important topic in computer programming. In fact, computers were invented to solve math and physics problems that were too difficult to do by hand. SPICE is essentially a specialized numerical analysis app.

Crudely, we could make a guess for the value of i_D and plug it into the equation. Most likely, our guess will be wrong and the left side of the equation will not equal zero. Based on the result, we can make a new guess and try again. We keep repeating until we zero in — converge — on the correct result. A well-written computer algorithm will take an initial guess and then automatically converge on the correct result after some number of iterations. The process stops when the change in the calculated result from one step to the next is smaller than the desired precision.

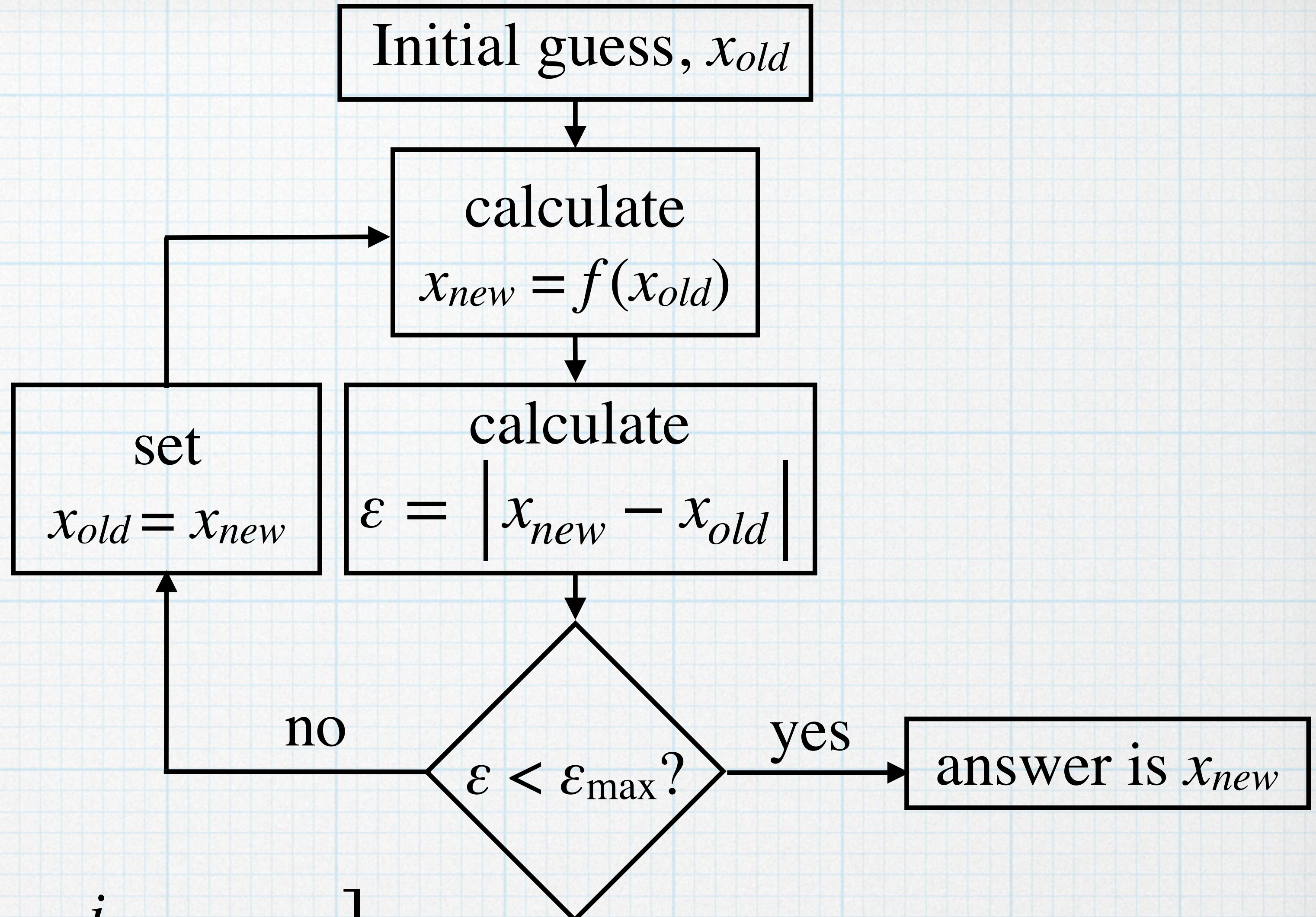
There are many algorithms for finding zeros of an equation. One method that we can apply here is *fixed-point iteration*. We start by re-writing the transcendental equation,

$$i_D = \frac{V_S}{R} - \frac{kT/q}{R} \ln \left[\frac{i_D}{I_S} + 1 \right]$$

$$= 1.5\,\text{mA} - \left(0.0258\,\text{mA}\right) \ln \left[\frac{i_D}{10^{-11}\,\text{mA}} + 1 \right]$$

The equation now has a general form of $x = f(x)$. A procedure for converging to the answer is depicted in the flow diagram on the next page.

Fixed-point iteration algorithm for solving $x = f(x)$. Choose a precision ε_{max} and start with an initial guess. Does not work for all functions $f(x)$, but when it does work, it tends to converge quickly.



$$i_D = 1.5\,\text{mA} - (0.0258\,\text{mA}) \ln\left[\frac{i_D}{10^{-11}\,\text{mA}} + 1\right] = 0$$

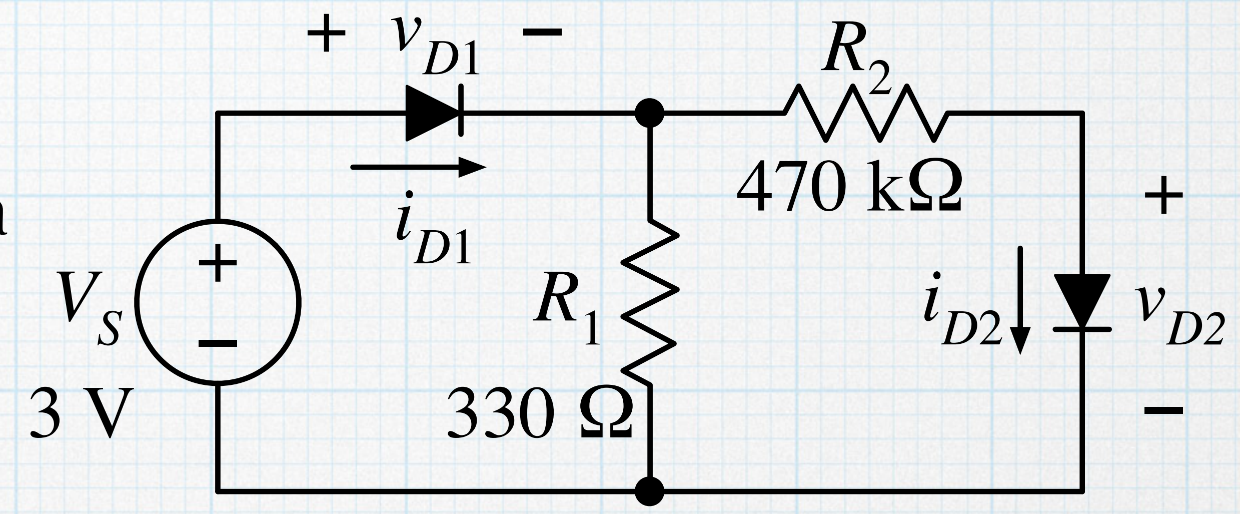
Applying the method to our diode equation, with an initial guess of 1.00 mA, gives the sequence shown. Within four iterations, the calculation has converged to 5 significant digits.

1.000000 mA	1st guess
0.846526 mA	
0.850825 mA	
0.850694 mA	
0.850698 mA	$= i_D$!!

Going back to the circuit, we can now calculate $v_D = 0.649$ V.

Although the non-linear nature of the diode caused some mathematical difficulties requiring numerical techniques, the analysis wasn't really that bad. In fact, the iteration technique was sort of interesting.

However, the difficulty escalates quickly when circuits become just a little bit more complicated. Consider the circuit at right — it doesn't look that bad.



Try node voltage (ground at the bottom, two node voltages v_{R1} and v_{D2}):

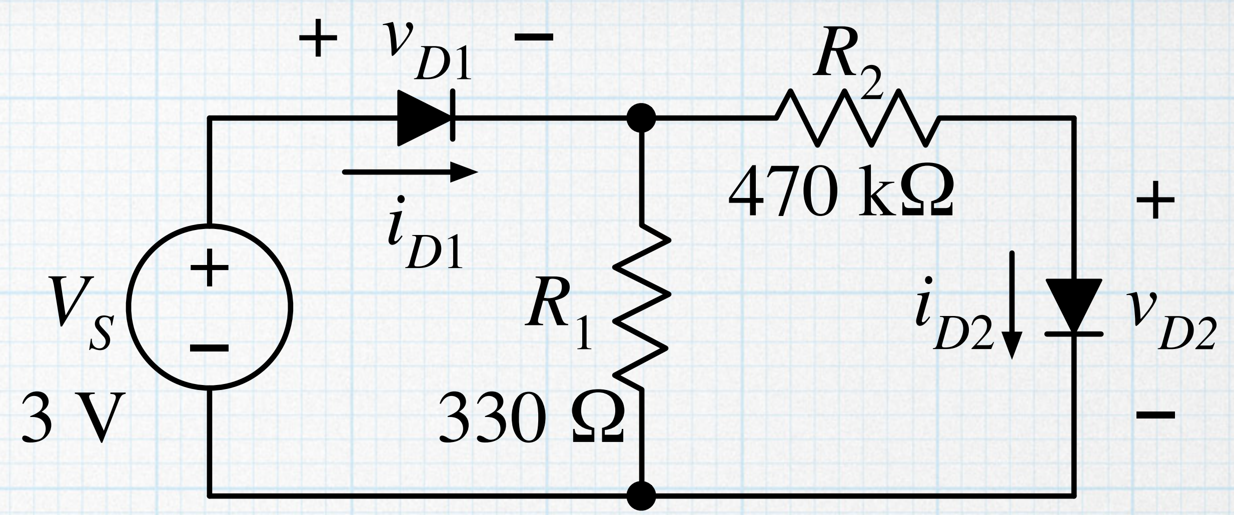
$$i_{D1} = i_{R1} + i_{R2} = \frac{v_{R1}}{R_1} + \frac{v_{R1} - v_{D2}}{R_2} \qquad\qquad i_{D2} = \frac{v_{R1} - v_{D2}}{R_2}$$

$$I_{S1}\left[\exp\left(\frac{v_{D1}}{kT/q}\right) - 1\right] = I_{S1}\left[\exp\left(\frac{V_S - v_{R1}}{kT/q}\right) - 1\right] = \frac{v_{R1}}{R_1} + \frac{v_{R1} - v_{D2}}{R_2}$$

$$I_{S2}\left[\exp\left(\frac{v_{D2}}{kT/q}\right) - 1\right] = \frac{v_{R1}}{R_1} + \frac{v_{R1} - v_{D2}}{R_2}$$

Two unknowns, related by two *non-linear* equations! Not good.

Maybe mesh currents would be better. We can relate the two currents i_{D1} and i_{D2}.



$$V_S - v_{D1} - v_{R1} = 0 \qquad V_S - v_{D1} - R_1\left(i_{D1} - i_{D2}\right) = 0$$

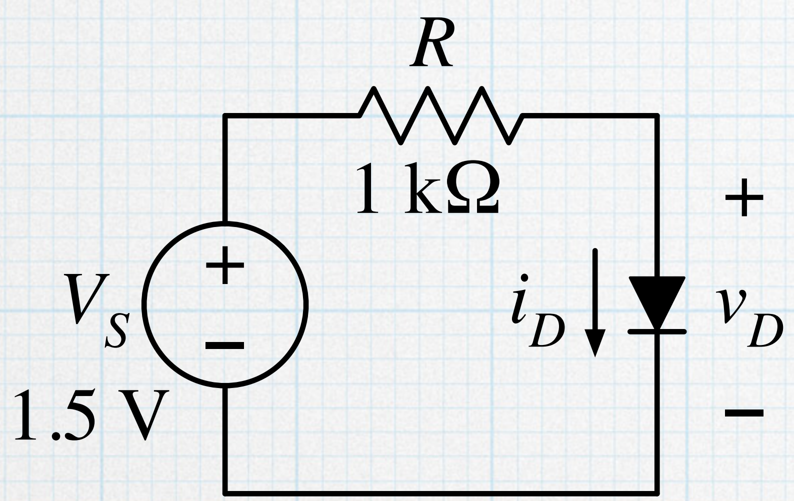
$$v_{R1} - v_{R2} - v_{D2} = 0 \qquad R_1\left(i_{D1} - i_{D2}\right) - R_2 i_{D2} - v_{D2} = 0$$

$$V_S - \frac{kT}{q}\ln\left[\frac{i_{D1}}{I_{S1}} + 1\right] - R_1\left(i_{D1} - i_{D2}\right) = 0$$

$$R_1\left(i_{D1} - i_{D2}\right) - R_2 i_{D2} - \frac{kT}{q}\ln\left[\frac{i_{D2}}{I_{S2}} + 1\right] = 0$$

Still two non-linear equations relating the two unknowns. There is no way around it. No other techniques are even applicable — no such thing as equivalent diodes, diode dividers, or diode transformations. Our best option is to look for some sort of short-cut.

Return to the earlier single-diode example. Solve for i_D and v_D exactly for a range of V_S values.



$$i_D = \frac{V_S}{R} - \frac{kT/q}{R} \ln\left[\frac{i_D}{I_S} + 1\right]$$

When the diode is reverse-biased ($V_S < 0$, so $v_D < 0$), it behaves essentially like an open circuit, $i_D \approx 0$.

When the diode is forward-biased ($V_S > 0$, so $v_D > 0$), its voltage is approximately the same in each case, $v_D \approx 0.7$ V.

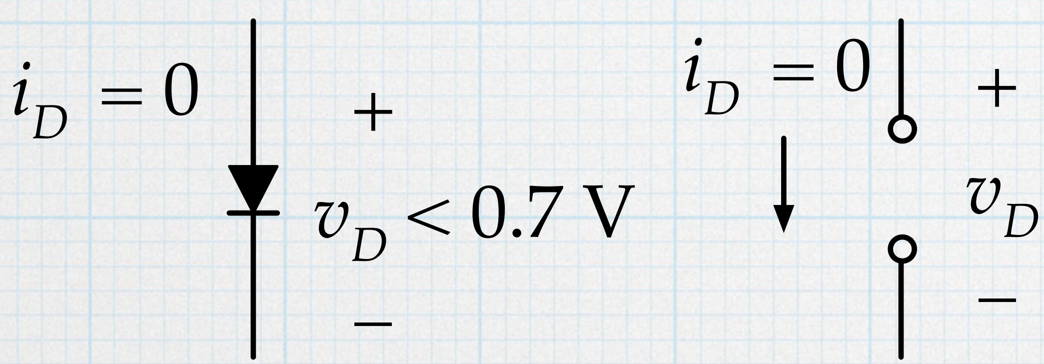
V_S (V)	v_D (V)	i_D (mA)
–10	–10	≈ 0
–8	–8	≈ 0
–6	–6	≈ 0
–4	–4	≈ 0
–2	–2	≈ 0
0	0	0
1	0.628	0.372
2	0.661	1.339
3	0.675	2.325
4	0.684	3.316
5	0.691	4.309
6	0.697	5.304
7	0.701	6.299
8	0.705	7.295
9	0.708	8.292
10	0.711	9.289

piecewise diode model

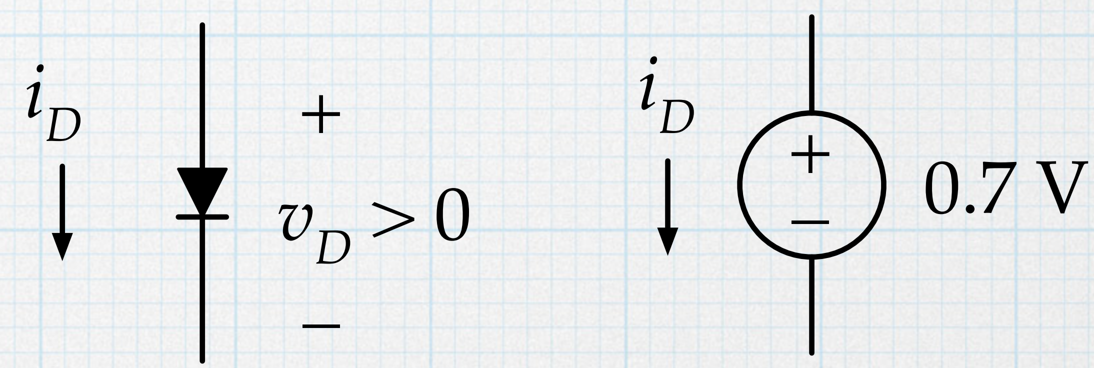
The results of the previous slide suggest an approximation:

- When reverse-biased, we treat the diode as if it is an open-circuit.
- When forward-biased, we treat it like an ideal source with a value of 0.7 V.

Reverse ($v_D < 0.7$ V)



Forward ($v_D = 0.7$ V)

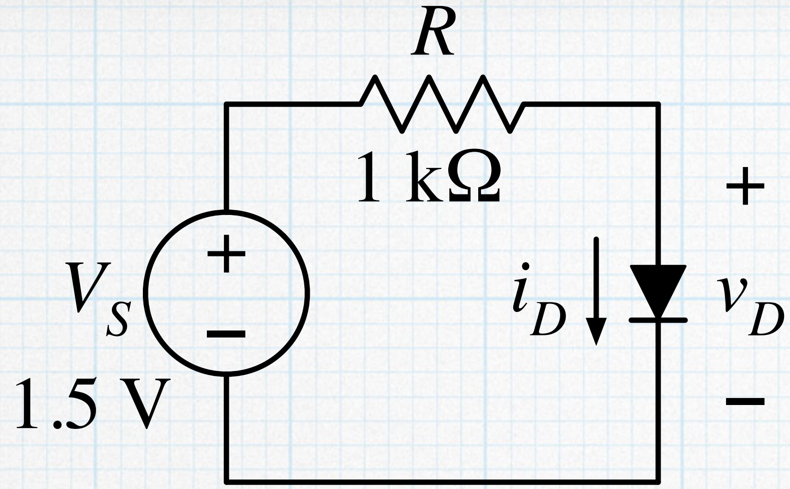


To use the models.

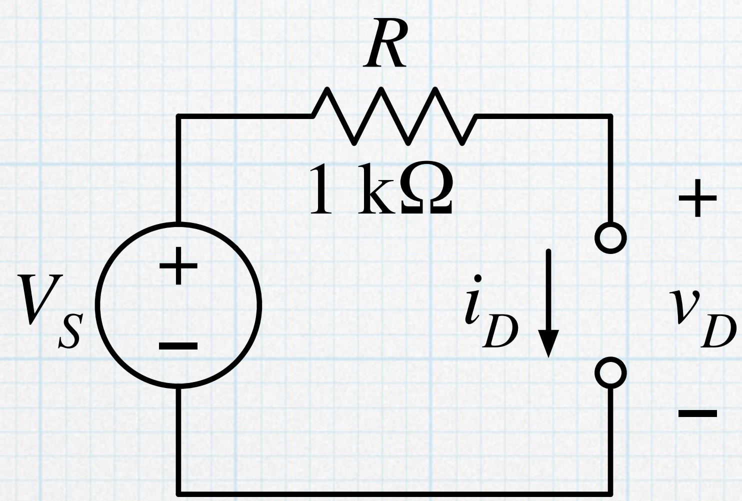
- Guess forward or reverse
- Insert the corresponding model
- Solve for voltage/current using model
- Check the result: for reverse, $v_D < 0$, for forward, i_D flows in correct direction

Note that the diode is NOT a voltage source. It is a passive device and does not provide power to the circuit. But it does behave as if it were a voltage source or battery that is absorbing power.

Use the piecewise-linear approximation.

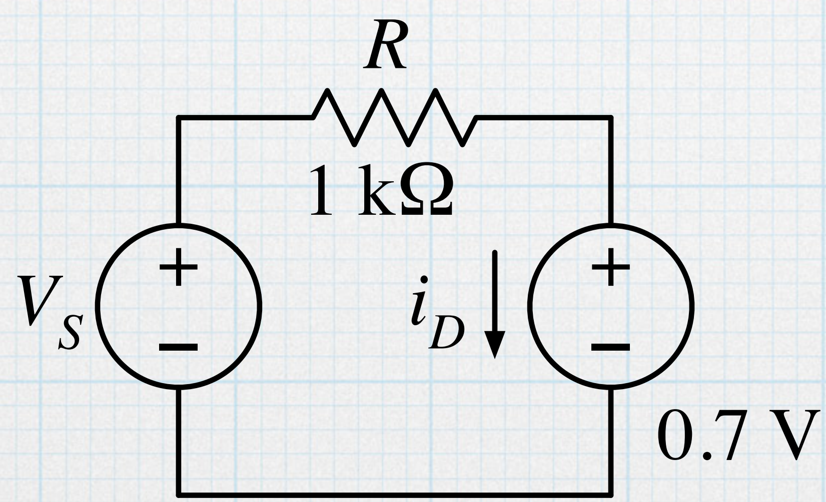


$V_S < 0.7$ V:
reverse bias.
$i_D = 0$.
$v_D = V_S$.



$V_S > 0.7$ V:
forward bias.
$v_D = 0.7$ V.

$$i_D = \frac{V_S - 0.7\text{ V}}{R}$$

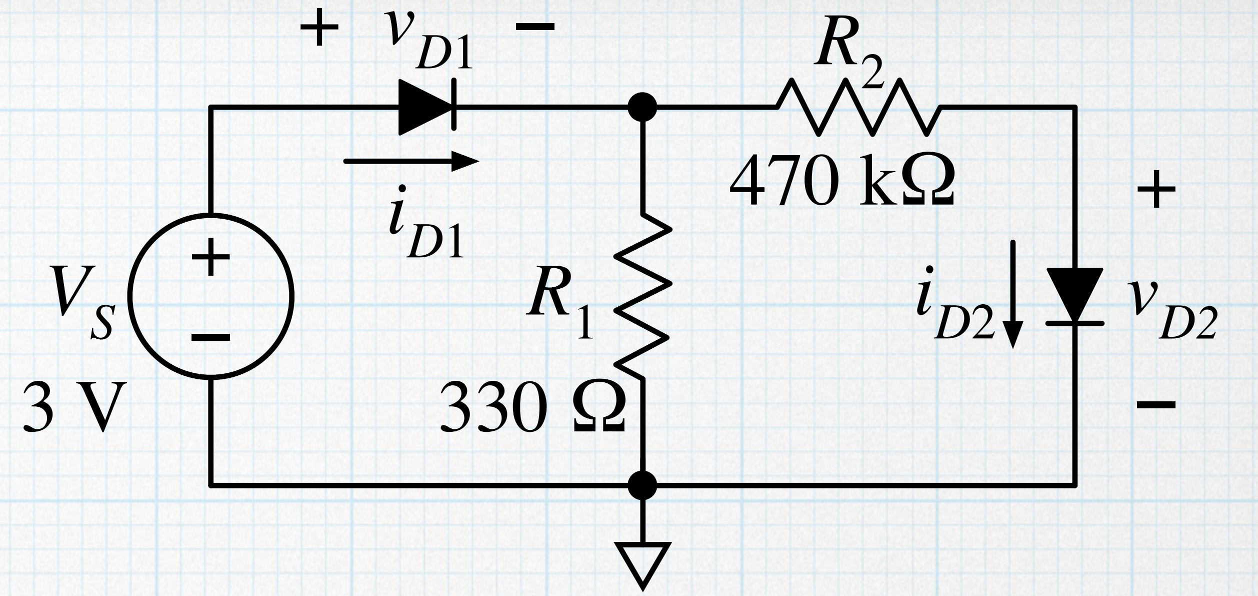


V_S (V)	v_D (V)	i_D (mA)
–10	–10	≈ 0
–8	–8	≈ 0
–6	–6	≈ 0
–4	–4	≈ 0
–2	–2	≈ 0
0	0	0
1	0.7	0.3
2	0.7	1.3
3	0.7	2.3
4	0.7	3.3
5	0.7	4.3
6	0.7	5.3
7	0.7	6.3
8	0.7	7.3
9	0.7	8.3
10	0.7	9.3

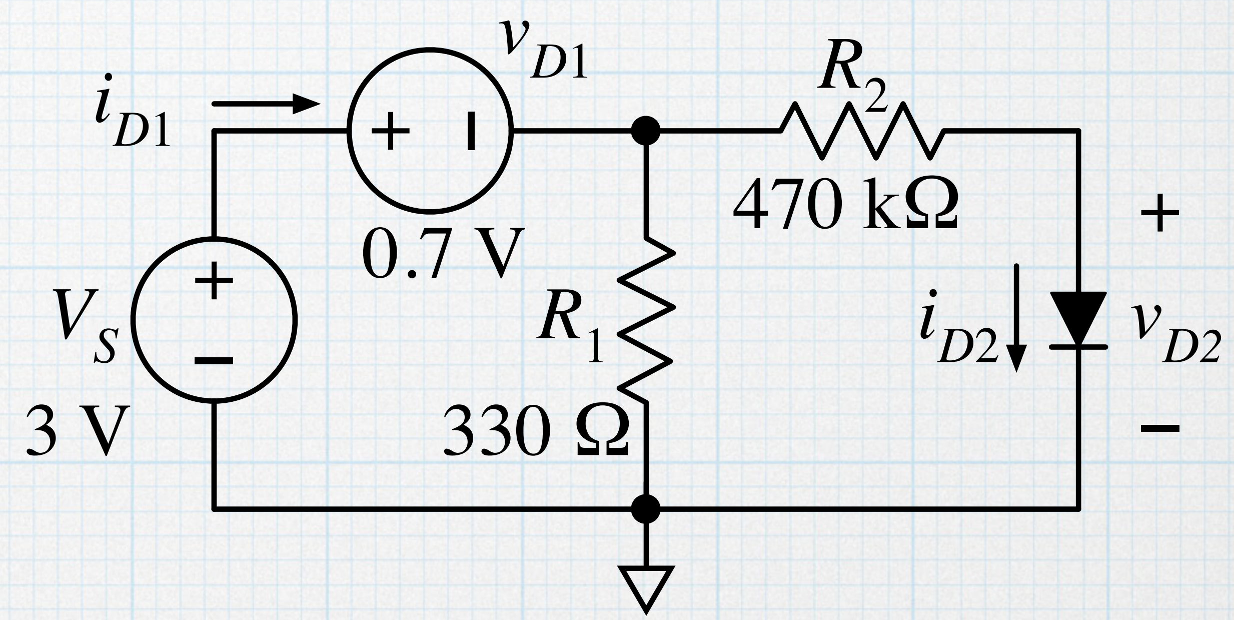
Results are quite close to the "exact" values, with a fraction of the effort.

Example

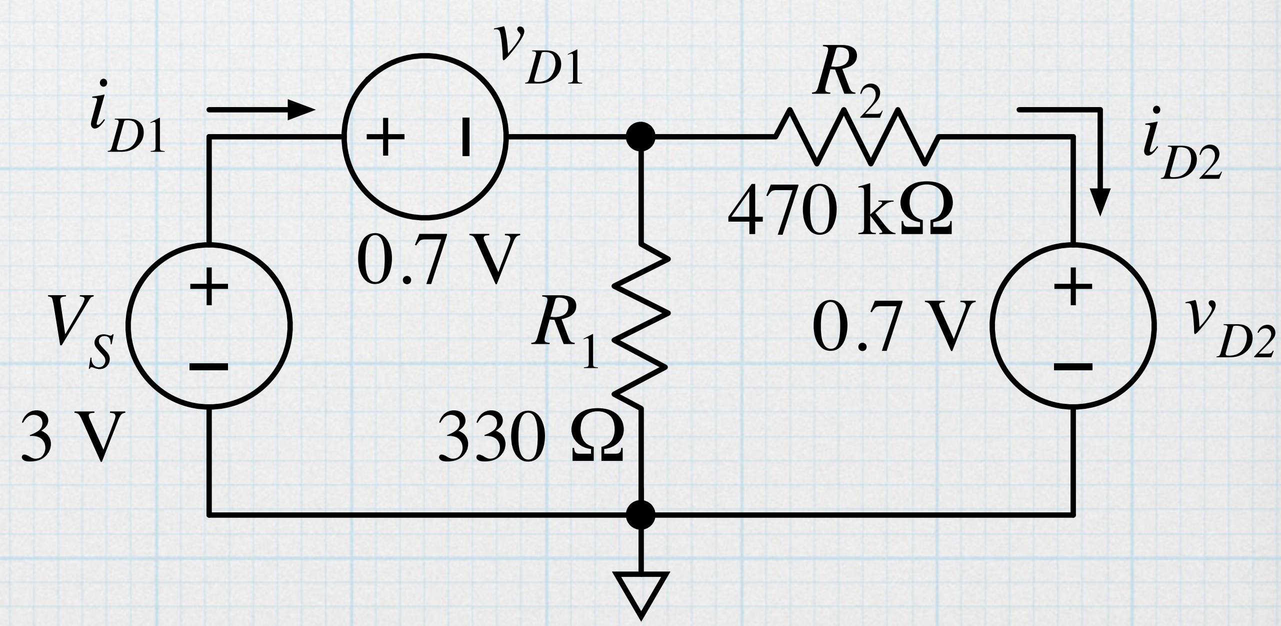
Let's apply the piecewise linear model (on-off model) to the two-diode circuit. Call the bottom node ground as a reference.

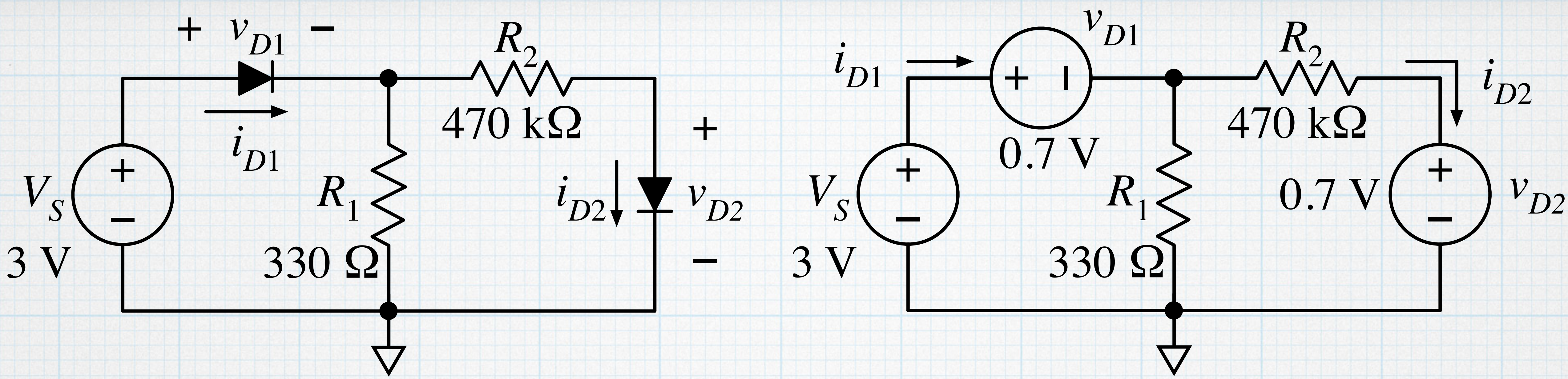


The first step is to choose a mode of operation (on or off) for each diode. We can say with some certainty that D1 is on. (If it were off, there would be no current anywhere in the circuit and all of the source voltage would drop across D1, making v_{D1} = 3 V. Which completely contradicts our guess that it was off.)



If D1 is on, then $v_{R1} = V_S - v_{D1} = 2.3$ V. Using the same argument above, we can surmise that D2 is also on. (If it were off, it would have 2.3 V across it — again, a contradiction.





Having replaced the diodes with fixed voltages, the circuit analysis is nearly trivial.

We already calculated $v_{R1} = V_S - v_{D1} = 2.3$ V.

Then, $i_{R1} = v_{R1} / R_1 = 6.97$ mA.

And $i_{D2} = i_{R2} = v_{R2} / R_1 = (v_{R1} - v_{D2}) / R_2 = 3.40$ mA.

Finally, $i_{D1} = i_{R1} + i_{D2} = 10.4$ mA

As a final check, we see that the two diode currents are both in the correct direction for diodes being forward-biased. This confirms that our initial “guesses” were correct. (In this case, we really did not have to guess — we had deduced from the outset that the diodes were on.)