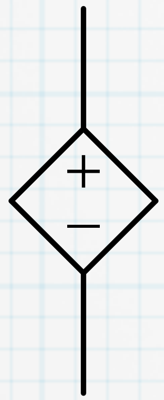
Dependent sources

As we begin to use simple circuits to model more complex circuit behavior, we need to add some items to our tool kit.

Dependent sources behave just like independent voltage and current sources, except that the voltage or current depends in some way on another voltage or current in the circuit.

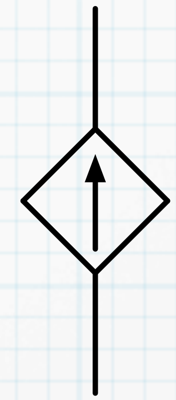
This seems a bit odd, but this behavior corresponds very closely to the way a number of interesting and useful electronic devices behave. We're not to try to get a detailed understanding of how these devices work internally – that's the subject for an electronics or semiconductor class. However, we can form a reasonable *model* of how the electronic devices behaves in a circuit by using dependent sources.

Dependent voltage source



$V_d = A v_1$

Dependent current source



$I_d = \beta i_1$

Here v_1 and i_1 are quantities defined somewhere else in the circuit, including the proper polarity or direction. These definitions must be included, or the circuit is not properly specified.

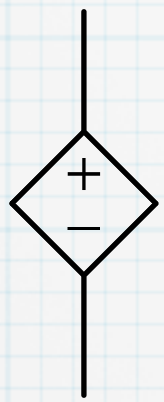
Note that the dependency factors A and β are dimensionless quantities.

For the voltage source above, since the voltage depends on another voltage, it is known as a *voltage-controlled voltage source* (*VCVS*).

Similarly, the current would be called a *current-controlled current source* (*CCCS*).

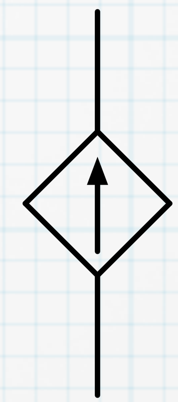
It is not necessary that the voltage source be dependent on another voltage or that the current source depend on another current.

Current-controlled voltage source (CCVS)



$V_d = \rho i_1$

voltage-controlled current source (VCCS)



$I_d = \gamma v_1$

Again, the controlling current, i_1 and the controlling voltage v_1 must be defined somewhere else in the circuit.

In these cases, the dependency factors will have units. For ρ, the units are **Ω**. This does not mean that ρ represents some type of resistor – it is simply the factor that relates the voltage to its controlling current. The units for γ must be siemans (S = A/V = $\mathbf{\Omega}^{-1}$).

Once the dependent source are located in circuit, along with the definitions for the controlling currents or voltages, then circuit analysis proceeds as always. Kirchoff's current and voltage laws still apply and all of the techniques derived from those still apply. In particular, voltage dividers, the node-voltage method, and the loop current technique are unchanged.

Source transformations must be used with caution. Since the dependent source in defined in terms of a particular voltage or current, you must be careful about changing the definitions – the overall circuit behavior must remain unchanged.

When using superposition, dependent sources *cannot* be removed. The dependent source must stay in place for all of the partial circuits you as consider each *independent* source in turn.

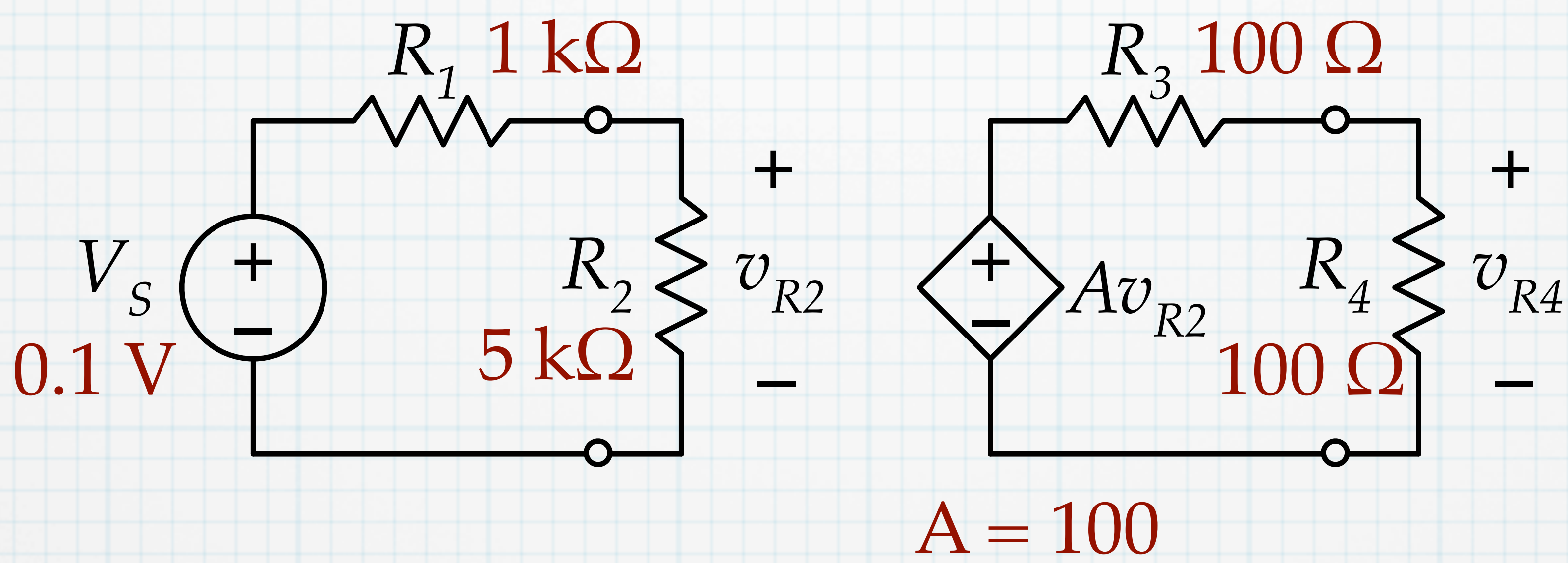
When doing Thevenin equivalents, you cannot remove the dependent sources when trying to determine the equivalent resistance using the *short-cut* method. Thus, when dependent sources are present, the short-cut technique become somewhat less useful.

As long as you remember those caveats for the source transformations, superposition, and Thevenin equivalents, everything that we've learned to this point can be applied to circuits with dependent sources.

Finally, in circuits with the dependent sources, energy and power may not balance in the manner that we have expected for circuits that we have seen up till now. This happens because the

Example 1 (amplifier)

The circuit below uses a voltage-dependent voltage source to approximate the behavior of amplifier. The amplifier model consists of the resistors R_2 and R_3 and the dependent source. (We will study amplifiers in more detail soon.) In the circuit, find v_{R4}.



Find an expression for v_{R4} using a voltage divider

$$v_{R4} = \frac{R_4}{R_4 + R_3}(Av_{R2}) = \frac{100\Omega}{100\Omega + 100\Omega}(100)\, v_{R2} = 50 v_{R2}$$

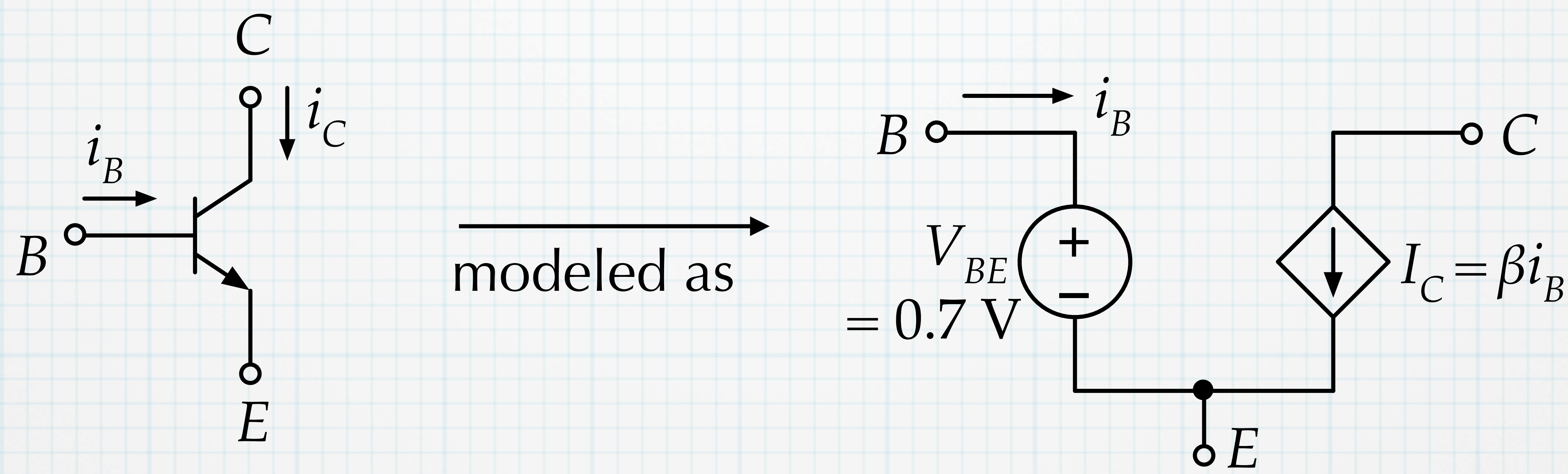
Need to find v_{R2}. Use a voltage divider.

$$v_{R2} = \frac{R_2}{R_2 + R_2} V_S = \frac{5\text{ k}\Omega}{1\text{ k}\Omega + 5\text{ k}\Omega}(0.1\text{ V}) = 0.0833\text{ V}$$

Substituting back: $v_{R4} = (50)(0.0833\text{ V}) = 4.17\text{ V}$.

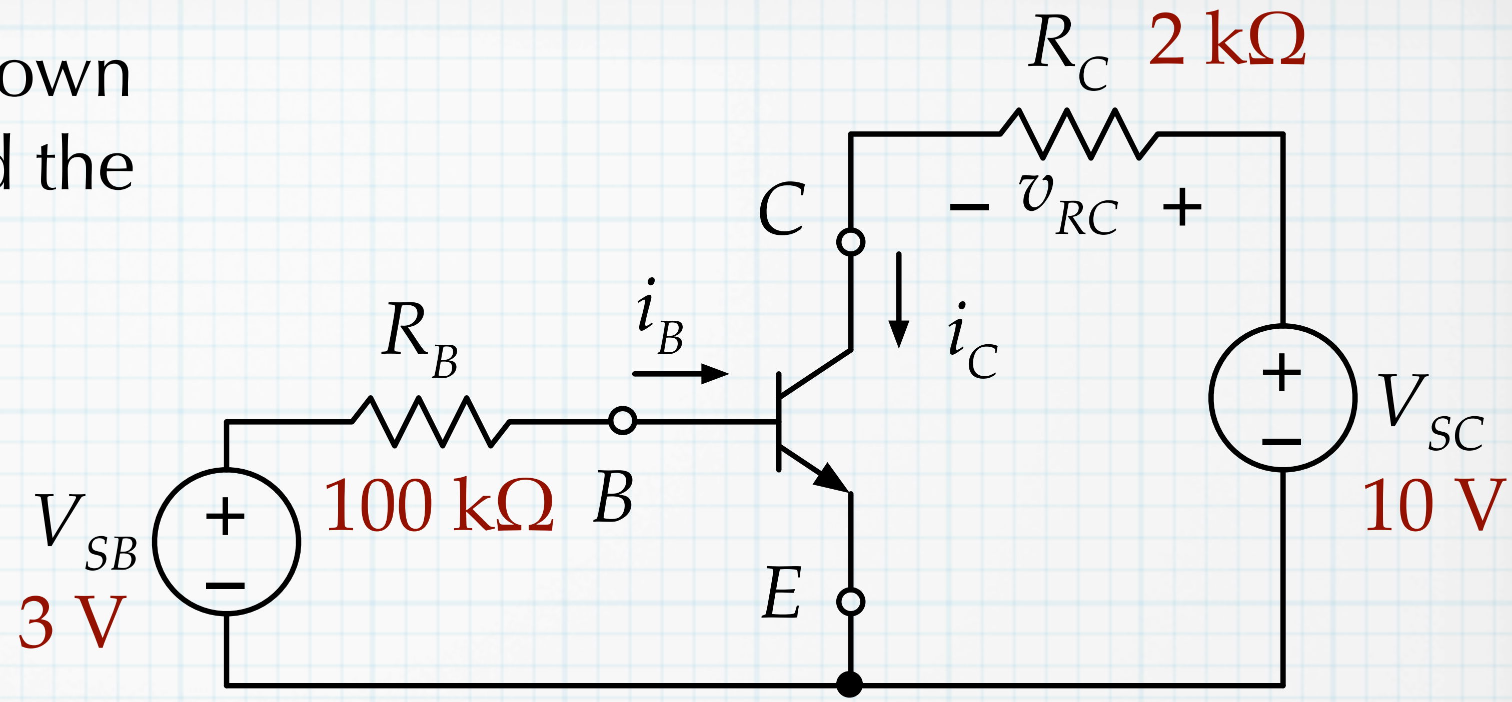
Example 2 (transistors)

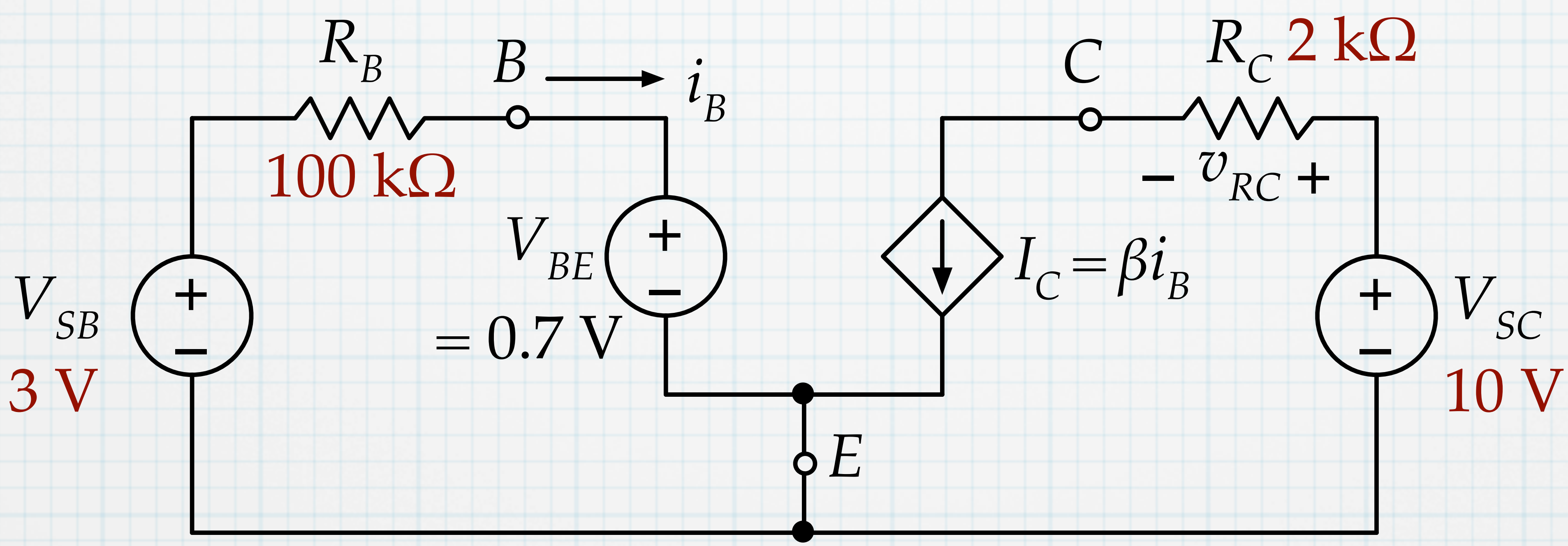
A bipolar junction transistor (BJT) is an extremely non-linear (but extremely useful!) circuit element that will studied in detail in EE 230 (and later).



Even though it is fundamentally non-linear, in one particular mode of operation it behaves in a somewhat linear fashion and can be modeled using linear components, as shown at right. The primary feature of the BJT in this mode is current gain – a small current (i_B) flowing in at one terminal leads to an "amplified" current flowing in another terminal.

A BJT is used in a circuit as shown at right. Use the circuit to find the resistor voltage v_{RC}. For the transistor $\beta = 100$.





Around the left-hand loop:
$V_{SB} - i_B R_B - V_{BE} = 0.$

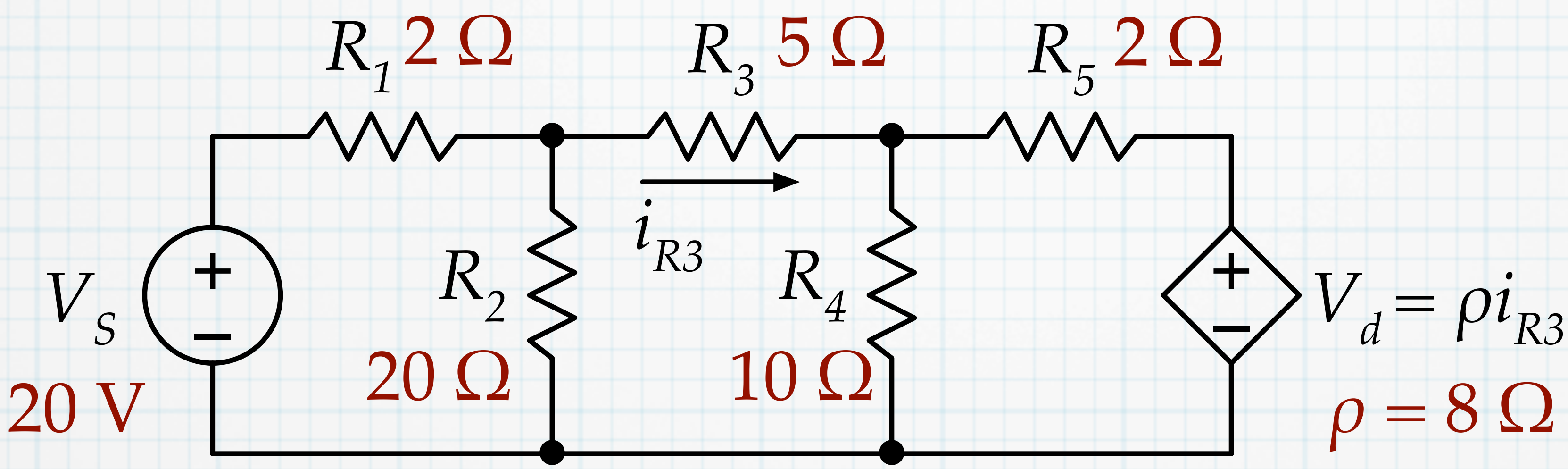
$$\frac{V_{SB} - V_{BE}}{R_B} = \frac{3\text{ V} - 0.7\text{ V}}{100\text{ k}\Omega} = 0.023\text{ mA}$$

On the right: $I_C = \beta i_B = (100)(0.023\text{ mA}) = 2.3\text{ mA}.$

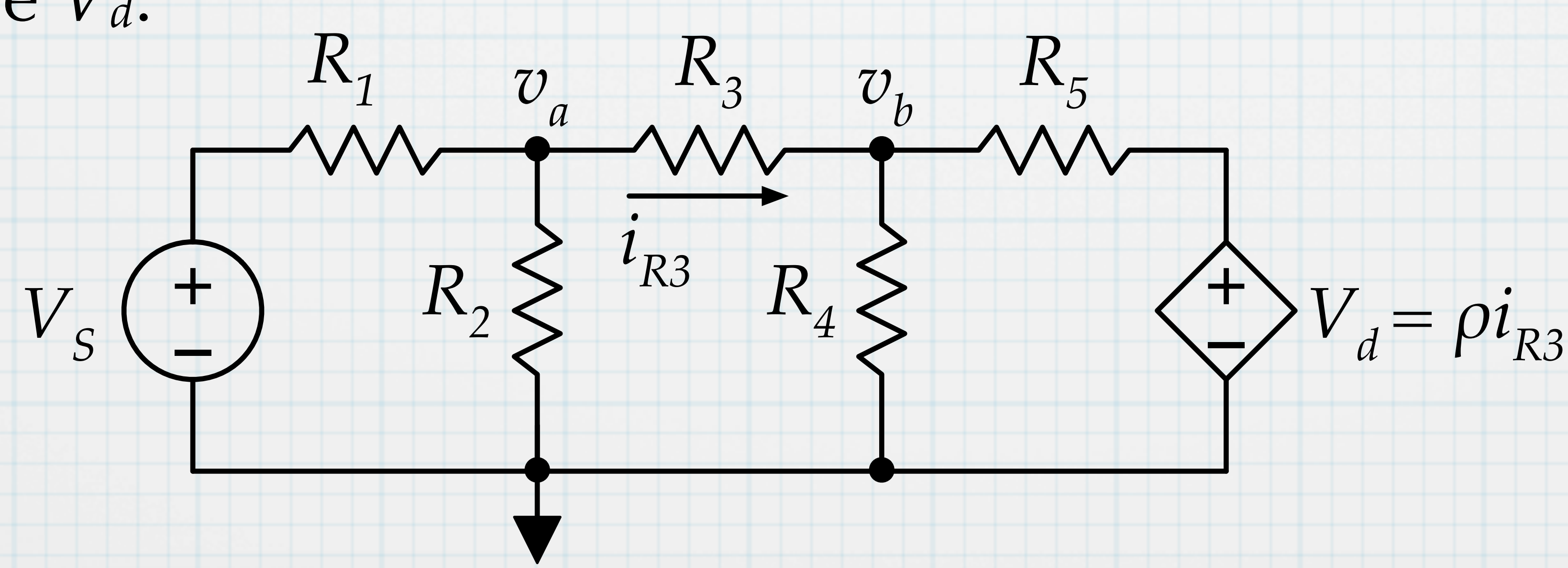
Then: $v_{RC} = R_C I_C = (2\text{ k}\Omega)(2.3\text{ mA}) = 4.6\text{ V}.$ So easy!

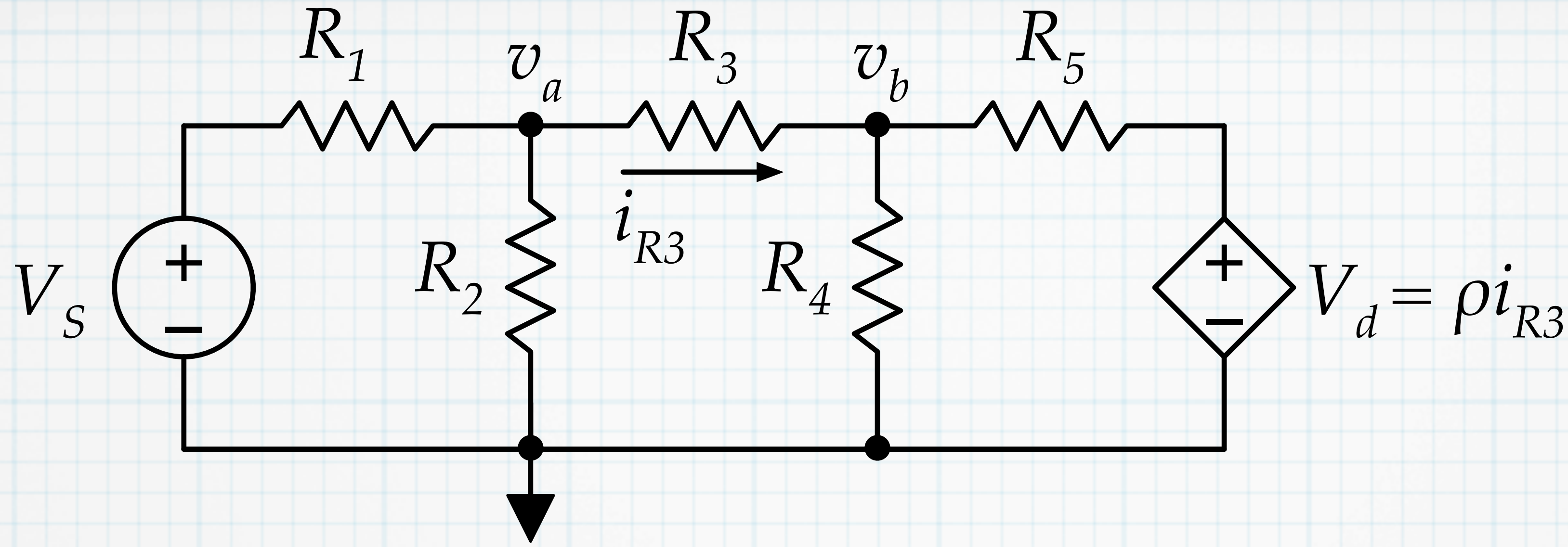
Example 3 (node-voltage)

Find the voltage across R_3 in the circuit below.



Use the node-voltage method. Define ground at the bottom and identify the unknown nodes. Initially, treat V_d *as if* it were known. We will write node equations for v_a and v_b, and then we add an *auxiliary* equation to handle V_d.





Write the node-voltage equations. (You should fill in the missing steps.)

$$\frac{V_S - v_a}{R_1} = \frac{v_a}{R_2} + \frac{v_a - v_b}{R_3} \qquad \frac{v_a - v_b}{R_3} + \frac{V_d - v_b}{R_5} = \frac{v_b}{R_4}$$

At this point, we don't know V_d, so these are two equations in three unknowns. However, we obtain a third (auxiliary) equation easily by using the definition for dependent source:

$$V_d = \rho i_{R3} = \rho \left[\frac{v_a - v_b}{R_3} \right]$$

Inserting the expression for V_d into the second node-voltage equation:

$$\frac{v_a - v_b}{R_3} + \frac{1}{R_5}\left(\rho\left[\frac{v_a - v_b}{R_3}\right] - v_b\right) = \frac{v_b}{R_4}$$

Along with the first node-voltage equation, this gives us two equations in the two unknowns.

$$\frac{V_S - v_a}{R_1} = \frac{v_a}{R_2} + \frac{v_a - v_b}{R_3}$$

Working out the algebra:

$$\left[1 + \frac{R_1}{R_2} + \frac{R_1}{R_3}\right] v_a - \frac{R_1}{R_3} v_b = V_S$$

$$-\left[1 + \frac{\rho}{R_5}\right] v_a + \left[1 + \frac{\rho}{R_5} + \frac{R_3}{R_5} + \frac{R_3}{R_4}\right] v_b = 0$$

Inserting numbers:

$$1.5 v_a - 0.4 v_b = 20\text{ V}$$

$$-5 v_a + 8 v_b = 0$$

Solving gives: v_a = 16 V and v_b = 10 V, and the voltage across R_3 is $v_a - v_b$ = 6 V.