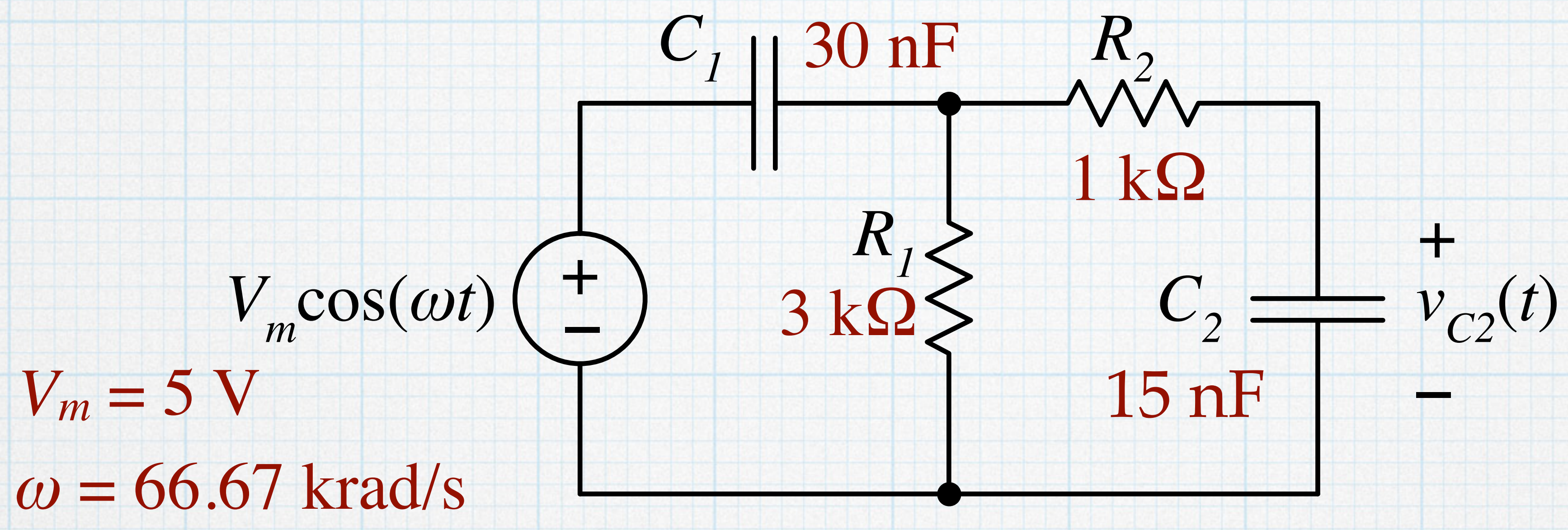
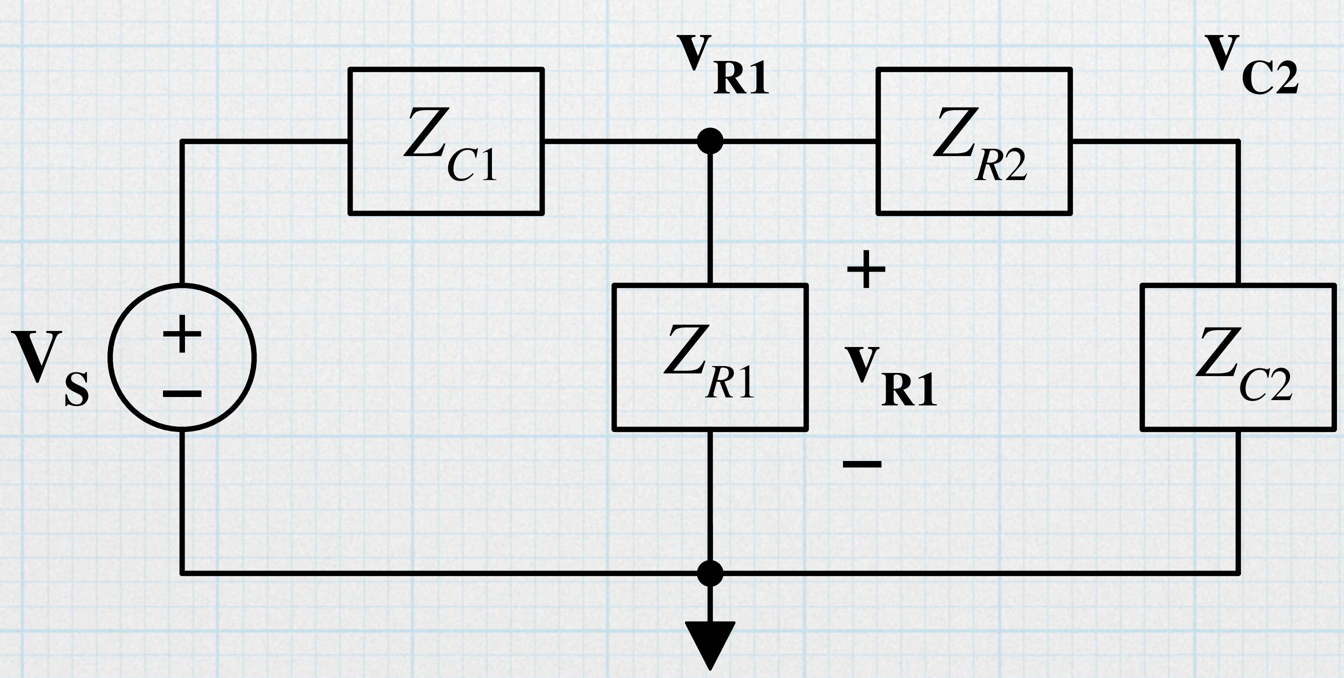
Example 7 - node voltage.

Same circuit as Example 4, but now solve it using node voltage.



Transform the circuit to its complex equivalent. The node voltages are $\mathbf{v}_{R1}$ and $\mathbf{v}_{C2}$.



Write the two node-voltage equations.

$$\frac{\mathbf{V_s} - \mathbf{v_{R1}}}{Z_{C1}} = \frac{\mathbf{v_{R1}} - \mathbf{v_{C2}}}{Z_{R2}} + \frac{\mathbf{v_{R1}}}{Z_{R1}}$$

$$\frac{\mathbf{v_{R1}} - \mathbf{v_{C2}}}{Z_{R2}} = \frac{\mathbf{v_{C2}}}{Z_{C2}}$$

Re-arrange a bit;

$$\left[1 + \frac{Z_{C1}}{Z_{R2}} + \frac{Z_{C1}}{Z_{R1}}\right] \mathbf{v_{R1}} - \frac{Z_{C1}}{Z_{R2}} \mathbf{v_{C2}} = \mathbf{V_s}$$

$$\mathbf{v_{R1}} = \left[1 + \frac{Z_{R2}}{Z_{C2}}\right] \mathbf{v_{C2}}$$

Substitute:

$$\left[1 + \frac{Z_{C1}}{Z_{R2}} + \frac{Z_{C1}}{Z_{R1}}\right] \left[1 + \frac{Z_{R2}}{Z_{C2}}\right] \mathbf{v_{C2}} - \frac{Z_{C1}}{Z_{R2}} \mathbf{v_{C2}} = \mathbf{V_s}$$

Substitute in for the impedances

$$\left[1+\frac{1}{j\omega R_2C_1}+\frac{1}{j\omega R_1C_1}\right]\left[1+j\omega R_2C_2\right]\mathbf{v_{C2}}-\frac{1}{j\omega R_2C_1}\mathbf{v_{C2}}=\mathbf{V_s}$$

Multiply all the terms and then gather real and imaginary parts. (Note that two of the terms cancel.) Grind it out.

$$\left[\left(1+\frac{C_2}{C_1}+\frac{R_2C_2}{R_1C_1}\right)+j\left(\omega R_2C_2-\frac{1}{\omega R_1C_1}\right)\right]\mathbf{v_{C2}}=\mathbf{V_s}$$

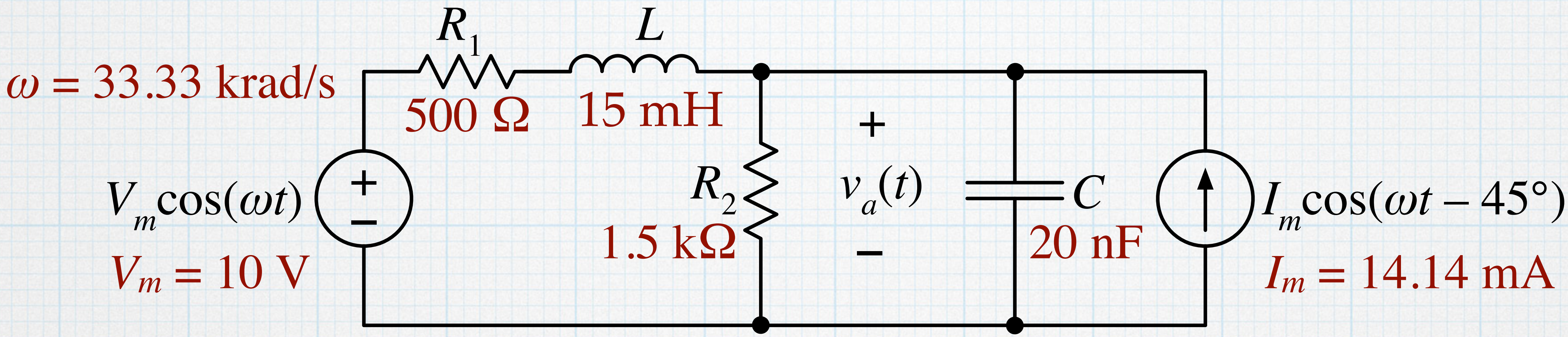
$$\left[1.667+j0.8333\right]\mathbf{v_{C2}}=\mathbf{V_s}$$

$$\left[\left(1.863\right)e^{26.6^\circ}\right]\mathbf{v_{C2}}=5\text{ V}$$

$$\mathbf{v_{C2}}=\left(2.68\text{ V}\right)e^{-j26.6^\circ}$$ Same as example 4.

Example 8 - node voltage.

Find the complex value of the capacitor voltage in the "two-source, two-impedance" circuit (Example 6) using the node voltage method.



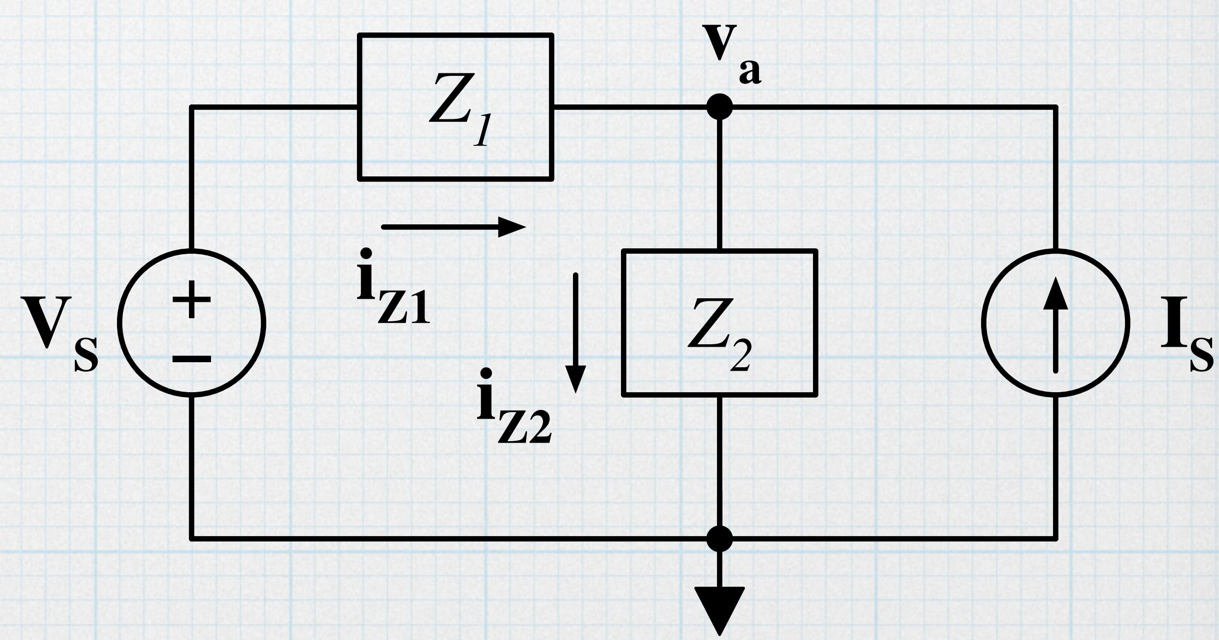
Transform to the complex form of the circuit. The circuit and the values are identical to Example 6.

$$\mathbf{V_S} = V_m e^{j0°} = 10\,\text{V}$$

$$\mathbf{I_S} = I_m e^{-j45°} = (14.14\,\text{mA})\, e^{-j45°}$$

$$Z_1 = (707.1\,\Omega)\, e^{j45°}$$

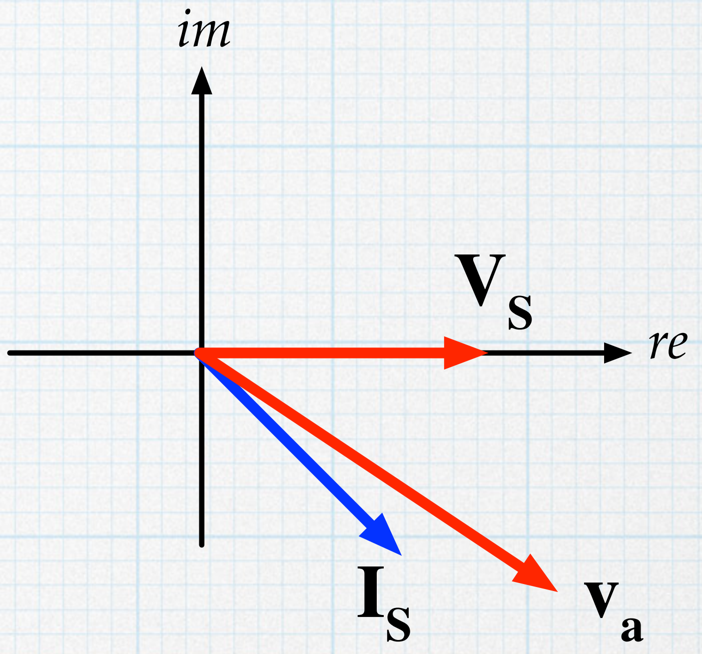
$$Z_2 = \left(1060.7\,\Omega\right) e^{-j45°}$$



$$\mathbf{i_{z1}} + \mathbf{I_S} = \mathbf{i_{z2}}$$

$$\frac{\mathbf{V_S} - \mathbf{v_a}}{Z_1} + \mathbf{I_S} = \frac{\mathbf{v_a}}{Z_2}$$

$$\mathbf{v_a} = \frac{\mathbf{V_S} + Z_1\mathbf{I_S}}{1 + \frac{Z_1}{Z_2}}$$



$$= \frac{10\,\text{V} + \left[(707.1\,\Omega)\,e^{j45^\circ}\right]\left[(14.14\,\text{mA})\,e^{-j45^\circ}\right]}{1 + \frac{(707.1\,\Omega)e^{j45^\circ}}{(1061\,\Omega)e^{-j45^\circ}}}$$

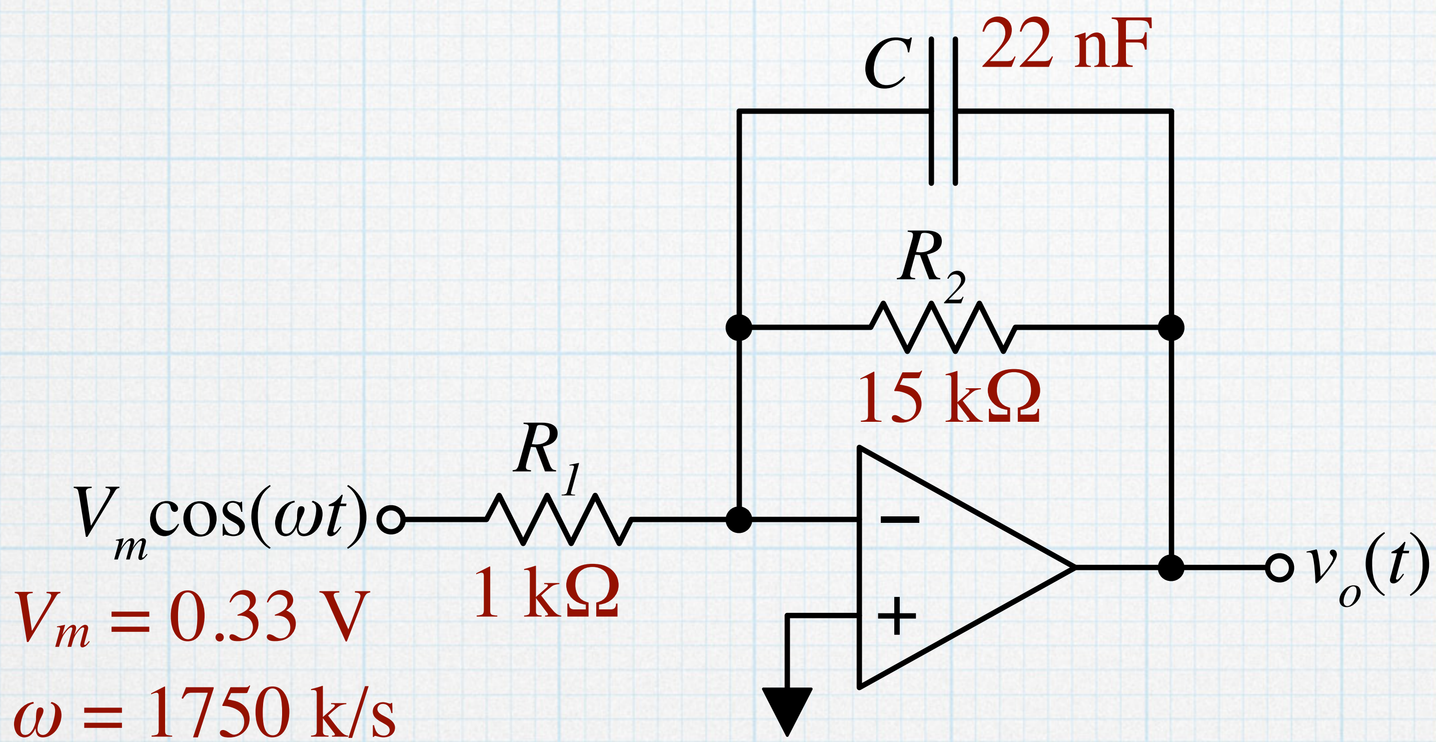
$$= \frac{10\,\text{V} + (10\,\text{V})\,e^{j0^\circ}}{1 + (0.667)\,e^{j90^\circ}} = \frac{20\,\text{V}}{1 + j0.667} = \frac{20\,\text{V}}{1.20e^{j33.7^\circ}}$$

$$\mathbf{v_a} = (16.6\,\text{V})\,e^{-j33.7^\circ}$$

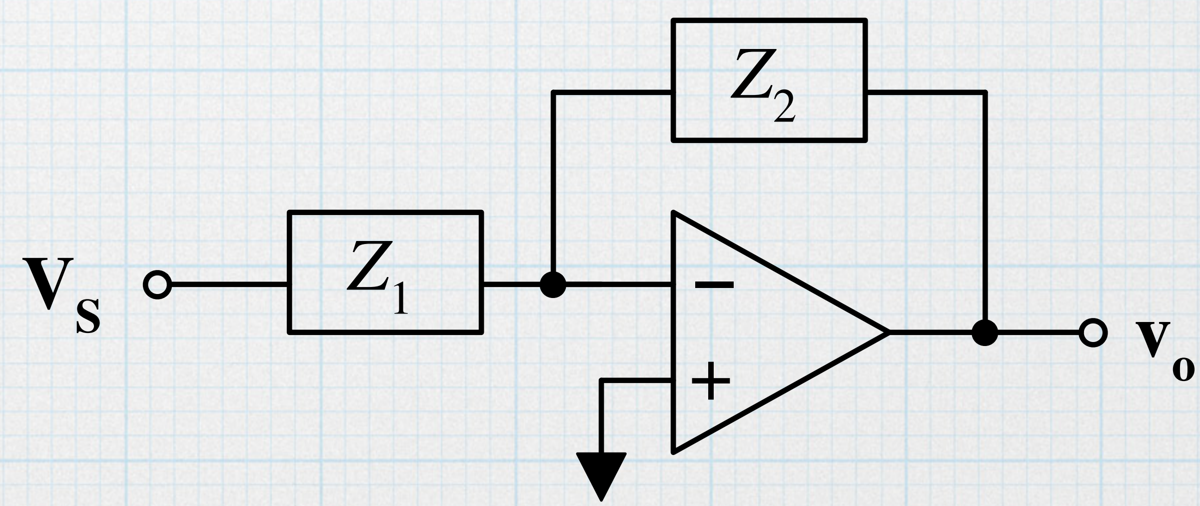
Identical to the result obtained in Example 6 using a source transformation.

Example 9 - node voltage.

Find the complex value of the output voltage in the op-amp circuit shown below. The op-amp is ideal.



The complex version of the circuit is shown. Using the properties of an ideal op amp with feedback, we can write a node-voltage equation at the inverting terminal:



$$\frac{\mathbf{V_S} - \mathbf{v_-}}{Z_1} = \mathbf{i_-} + \frac{\mathbf{v_-} - \mathbf{v_o}}{Z_2}$$

$$\mathbf{i_-} = 0$$

$$\mathbf{v_-} = \mathbf{v_+} = 0$$

$$\frac{\mathbf{v_S}}{Z_1} = -\frac{\mathbf{v_o}}{Z_2}$$

$$\mathbf{v_o} = -\frac{Z_2}{Z_1}\mathbf{v_S}$$

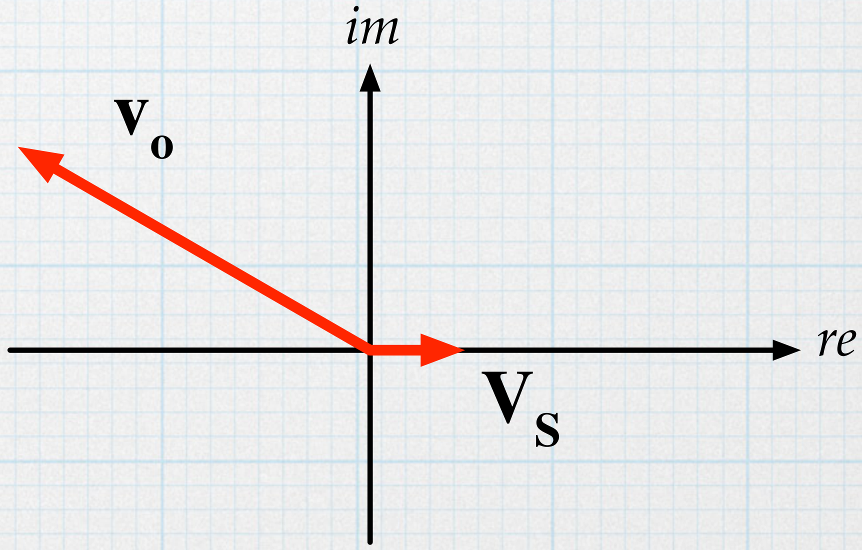
$$Z_2 = R_2 \Big\| \left(\frac{1}{j\omega C}\right)$$

$$= \frac{R_2}{1 + j\omega R_2 C} = \frac{15\,\mathrm{k\Omega}}{1 + j0.577}$$

$$Z_1 = R_1 = 1\,\mathrm{k\Omega}$$

$$\mathbf{v_o} = -\frac{15}{1 + j0.577}(0.33\ \mathrm{V}) = \frac{5\ \mathrm{V}}{1.155 e^{j30^\circ}}$$

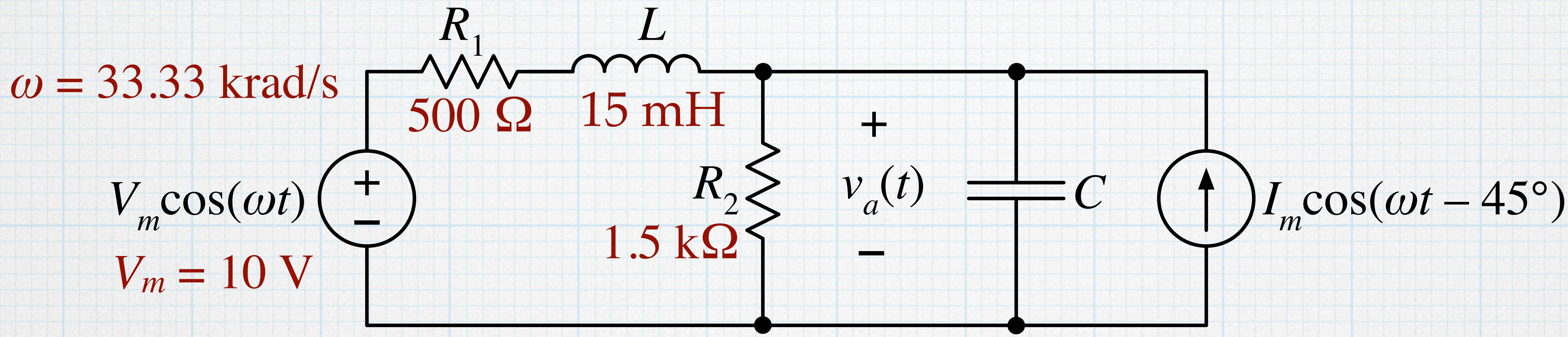
$$\mathbf{v_o} = -(4.33\ \mathrm{V})\, e^{-j30^\circ} = (4.33\ \mathrm{V})\, e^{j150^\circ}$$



Part of the phase shift is due to the reactance of the capacitor and part is due to the inverting nature of the op-amp.

Example 11 - mesh current.

Find the complex value of the capacitor voltage in the "two-source, two-impedance" circuit (Example 6) using the mesh-current method.

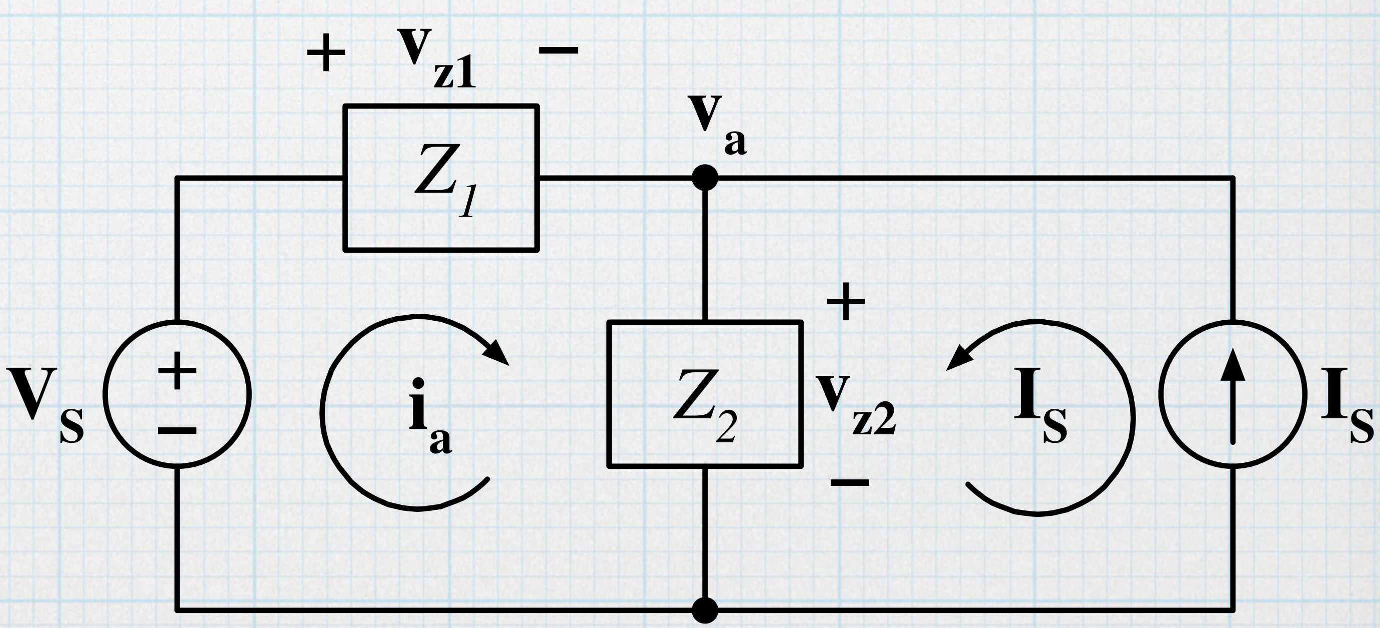


Transform to the complex form of the circuit. The circuit and the values are identical to Example 6. Define the two mesh currents. Of course, the right-hand current is known.

Around the left-hand mesh:

$$\mathbf{V_S} - \mathbf{v_{Z1}} - \mathbf{v_{Z2}} = 0$$

$$\mathbf{V_S} - \mathbf{i_a} Z_1 - \left(\mathbf{I_S} + \mathbf{i_a}\right) Z_2 = 0$$



$$\mathbf{i_a} = \frac{\mathbf{V_S} - \mathbf{I_S}Z_2}{Z_1 + Z_2}$$

$$= \frac{10\,\text{V} - \left[(14.14\,\text{mA})\,e^{-j45^\circ}\right]\left(1060.7\,\Omega\right)e^{-j45^\circ}}{\left(500\,\Omega + j500\,\Omega\right) + \left(750\,\Omega - j750\,\Omega\right)}$$

$$= \frac{10\,\text{V} + j15\,\text{V}}{1250\,\Omega - j250\,\Omega}$$

Be careful with the sign of the imaginary part.

$$= \frac{(18.03\,\text{V})\,e^{j56.3^\circ}}{(1274.8\,\Omega)\,e^{-j11.3^\circ}}$$

$$= (14.14\,\text{mA})\,e^{j67.6^\circ}$$

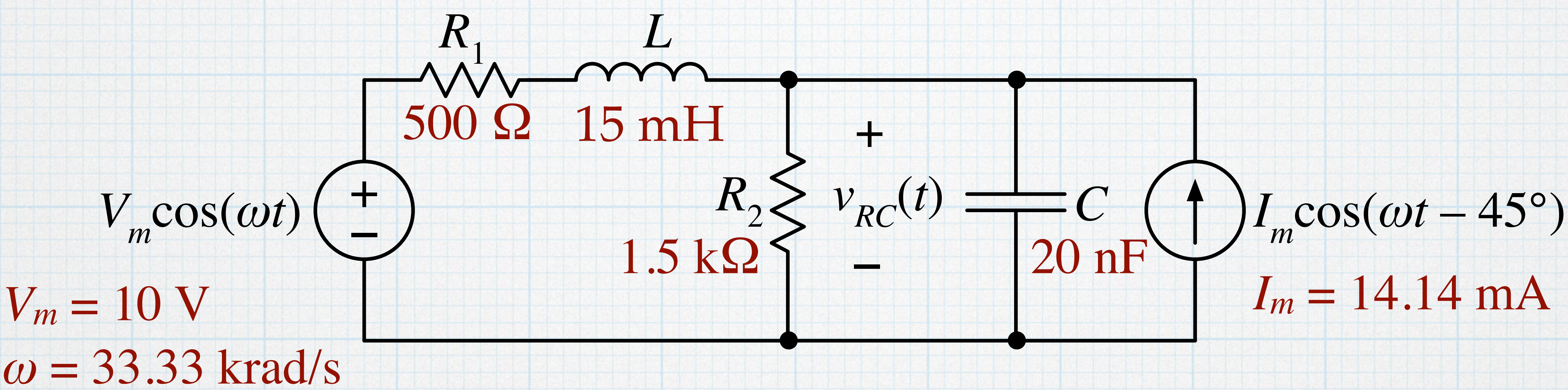
$$\mathbf{v_a} = \left(\mathbf{i_a} + \mathbf{I_S}\right)Z_2$$

$$= \left[(14.14\,\text{mA})\,e^{j67.6^\circ} + (14.14\,\text{mA})\,e^{-j45^\circ}\right]\left[\left(1060.7\,\Omega\right)e^{-j45^\circ}\right]$$

$$= \left(16.6\,\text{V}\right)e^{-j33.7^\circ}$$ Same.

Example 13 - superposition.

Find the complex value of the capacitor voltage in the "two-source, two-impedance" circuit using the superposition method.



$$\mathbf{v}_\mathbf{a}' = \frac{Z_2}{Z_2 + Z_1}\mathbf{V_S}$$

$$\mathbf{v}_\mathbf{a}' = \frac{(1061\,\Omega)\,e^{-j45°}}{(1061\,\Omega)\,e^{-j45°} + (707\,\Omega)\,e^{j45°}}(10\,\text{V})$$

$$Z_1 = (707.1\,\Omega)\,e^{j45°} \qquad Z_2 = (1060.7\,\Omega)\,e^{-j45°}$$

$$\mathbf{v}_\mathbf{a}' = \frac{Z_2}{Z_2 + Z_1}\mathbf{V_S}$$

$$\mathbf{v}_\mathbf{a}' = \frac{\left(1060.7\,\Omega\right) e^{-j45^\circ}}{\left(750\,\Omega - j750\,\Omega\right) + \left(500\,\Omega + j500\,\Omega\right)}(10\,\text{V})$$

$$= \frac{\left(1060.7\,\Omega\right) e^{-j45^\circ}}{1250\,\Omega - j250\,\Omega}(10\,\text{V})$$

$$= \frac{\left(1060.7\,\Omega\right) e^{-j45^\circ}}{(1274.8\,\Omega)\, e^{-j11.3^\circ}}(10\,\text{V})$$

$$= (8.32\,\text{V})\, e^{-j33.7^\circ}$$

$$\mathbf{v}_\mathbf{a}'' = \left(Z_1 \| Z_2\right)(\mathbf{I_S})$$

$$= \left[(588.3\,\Omega)\, e^{+j11.3^\circ}\right]\left[(14.14\,\text{mA})\, e^{-j45^\circ}\right]$$

$$= (8.32\,\text{V})\, e^{-j33.7^\circ}$$

The parallel combination was calculated in the source-transformation version (Example 6) of this problem. We re-use the result here.

$$\mathbf{v}_\mathbf{a} = \mathbf{v}_\mathbf{a}' + \mathbf{v}_\mathbf{a}''$$

$$= (8.32\,\text{V})\, e^{-j33.7^\circ} + (8.32\,\text{V})\, e^{-j33.7^\circ}$$

$$= (16.6\,\text{V})\, e^{-j33.7^\circ}$$

Same.