Direct current / alternating current (DC / AC)

The types of sources used in a circuit determine everything about the currents and voltages that we see in the circuit.

DC → does NOT change with time.

DC sources lead to circuit current, voltage, and power that are constant – unchanging with time.

There a numerous applications for DC circuits, but mostly used to supply power to electronic devices.

AC → Everything else, i.e anything that does change with time.

sinusoids (power distribution, communications & signal processing)

square waveforms (digital logic, communications)

triangle waveforms

Sinusoids (sines and cosines)

$$V_S(t) = V_a \sin(\omega t)$$

V_a → amplitude

ω → angular frequency

T → period (seconds)

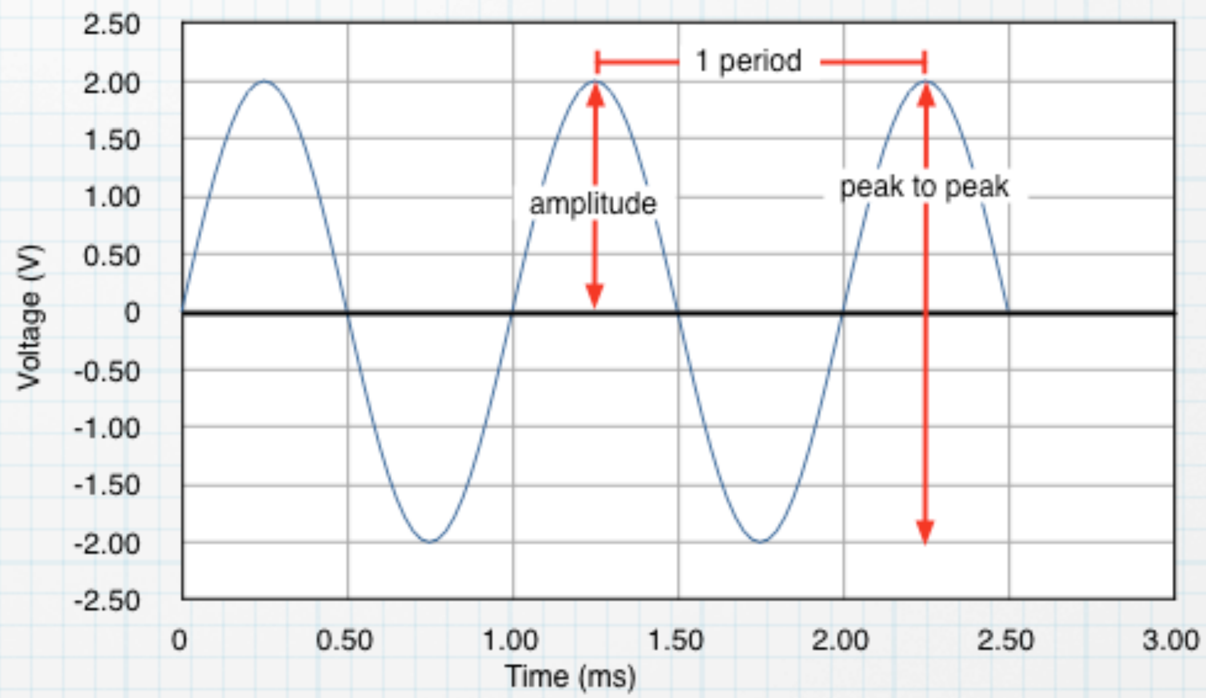
$f = T^{-1}$ → period (s^{-1} or hertz, Hz)

$\omega = 2\pi f$ → angular frequency (rad/s)

$$V_S(t) = V_a \sin\left(\frac{2\pi}{T}t\right)$$

$$= V_a \sin(2\pi f t)$$

$$= V_a \sin(\omega t)$$



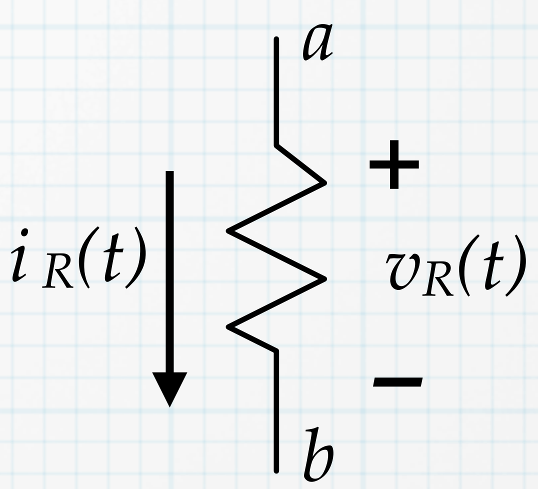
Cosine function is equally valid.

$$V_S(t) = V_a \cos\left(\frac{2\pi}{T}t\right) = V_a \cos(2\pi f t) = V_a \cos(\omega t)$$

Sinusoidal power in resistors

Consider a resistor with a voltage that is varying sinusoidally:

$$v_R\left(t\right) = V_a \sin\left(\omega t\right)$$



The current also varies sinusoidally:

$$i_R\left(t\right) = \frac{v_R\left(t\right)}{R} = \frac{V_a}{R} \sin\left(\omega t\right)$$

The dissipated power also varies with time:

$$P_R = v_R\left(t\right) i_R\left(t\right) = \frac{V_a^2}{R} \sin^2\left(\omega t\right)$$

Instantaneous power – always positive!

Average power

Find the average power delivered to the resistor is a straight-forward exercise in integration. Integrate over one full period (or an integral number of periods) and divide by the time.

$$P_{avg} = \frac{1}{T}\int_0^T P(t)\,dt$$

$$= \frac{1}{T}\int_0^T \frac{V_a^2}{R}\sin^2(\omega t)\,dt$$

$$= \frac{1}{T}\frac{V_a^2}{R}\int_0^T \left[\frac{1}{2} - \frac{1}{2}\sin(2\omega t)\right] dt$$

$$= \frac{1}{T}\frac{V_a^2}{R}\frac{1}{2}\left[\underbrace{\int_0^T dt}_{= T} - \underbrace{\int_0^T \sin(2\omega t)\,dt}_{= 0}\right]$$

$$= \frac{1}{T}\frac{V_a^2}{R}\frac{1}{2}[T] \quad = \frac{V_a^2}{2R} \qquad P_{avg} = \frac{V_a I_a}{2}$$

RMS values

To make it easy to compute powers in sinusoidal situations, we can define the "RMS amplitude". (root-mean-square)

$$P_{avg} = \frac{V_a I_a}{2} = \left(\frac{V_a}{\sqrt{2}}\right)\left(\frac{I_a}{\sqrt{2}}\right)$$

Define: $V_{RMS} = \frac{V_a}{\sqrt{2}}$ $\quad I_{RMS} = \frac{I_a}{\sqrt{2}}$

Then: $P_{avg} = V_{RMS} I_{RMS}$

$$P_{avg} = \frac{V_a^2}{2R} = \frac{V_{RMS}^2}{R}$$

$$P_{avg} = \frac{I_a^2}{2} R = I_{RMS}^2 R$$

Using RMS values makes the power equations for resistors identical to the DC case.

RMS values

Calculating RMS voltage or current directly: square it, find the average (mean), and take the square-root.

$$V_{RMS} = \sqrt{\frac{1}{T}\int_0^T v^2(t)\,dt}$$

$$I_{RMS} = \sqrt{\frac{1}{T}\int_0^T i^2(t)\,dt}$$

Can find the RMS for any voltage or current in a circuit (not just sources) and use it for power calculations.

To help denote RMS quantities in problems, in EE 201, we will append "RMS" as a subscript on the units.

examples: $v_{r2} = 3.6\ \mathrm{V_{RMS}}$ or $i_s = 7\ \mathrm{A_{RMS}}$.

RMS values

sinusoid: $v(t) = V_a \cos(\omega t)$

$$V_{RMS} = \sqrt{\frac{1}{T}\int_0^T V_a^2 \cos^2(\omega t)\, dt}$$

$$= \sqrt{\frac{V_a^2}{T}\int_0^T \left[\frac{1}{2} + \frac{1}{2}\cos(2\omega t)\right] dt}$$

(= T/2)

$$= \sqrt{\frac{V_a^2}{2}} = \frac{V_a}{\sqrt{2}}$$

DC: $v(t) = V_{DC}$

$$V_{RMS} = \sqrt{\frac{1}{T}\int_0^T V_{DC}^2 dt} = \sqrt{\frac{1}{T} V_{DC}^2 T} = V_{DC}$$

RMS in the lab

multi-meter: In AC measurement mode, the values given are *always* RMS units.

function generator: Sinusoidal voltages can be described in terms of either peak-to-peak or RMS units. It's your choice, but be sure that you know which you are using. (Reminder: Don't forget about the "high-Z" setting on the function generator.)

Oscilloscope: Again, your choice. It will give values in peak-to-peak or RMS. Make sure that you know what you are reading.

In general, when measuring values, the multi-meter will probably be more accurate than the number that come off the oscilloscope. Not always true (depends on the scope and the meter), but usually the case.